

-NoValue-

syn.1 Terms and Fórmulas

sol:syn:frm:
sec

Like in first-order logic, expressions of second-order logic are built up from a basic vocabulary containing *variáveis*, *símbolos de constante*, *símbolos de predicado* and sometimes *símbolos de função*. From them, together with logical connectives, quantifiers, and punctuation symbols such as parentheses and commas, *terms* and *fórmulas* are formed. The difference is that in addition to variables for objects, second-order logic also contains variables for relations and functions, and allows quantification over them. So the logical symbols of second-order logic are those of first-order logic, plus:

1. A *infinitos enumeráveis* set of second-order relation *variáveis* of every arity n : $V_0^n, V_1^n, V_2^n, \dots$
2. A *infinitos enumeráveis* set of second-order function *variáveis*: $u_0^n, u_1^n, u_2^n, \dots$

Just as we use x, y, z as meta-variables for first-order variables v_i , we'll use X, Y, Z , etc., as metavariables for V_i^n and u, v , etc., as meta-variables for u_i^n .

The non-logical symbols of a second-order language are specified the same way a first-order language is: by listing its *símbolos de constante*, *símbolos de função*, and *símbolos de predicado*.

explanation

In first-order logic, the *predicado de identidade* $=$ is usually included. In first-order logic, the non-logical symbols of a language $\mathcal{L}$ are crucial to allow us to express anything interesting. There are of course *sentenças* that use no non-logical symbols, but with only $=$ it is hard to say anything interesting. In second-order logic, since we have an unlimited supply of relation and function variables, we can say anything we can say in a first-order language even without a special supply of non-logical symbols.

Definição syn.1 (Second-order Terms). The set of *second-order terms* of $\mathcal{L}$, $\text{Trm}^2(\mathcal{L})$, is defined by adding to ?? the clause

1. If u is an n -place function variable and $t_1, \dots, t_n$ are terms, then $u(t_1, \dots, t_n)$ is a term.

So, a second-order term looks just like a first-order term, except that where a first-order term contains a *símbolo de função* f_i^n , a second-order term may contain a function variable u_i^n in its place.

explanation

Definição syn.2 (Second-order fórmula). The set of *second-order fórmulas* $\text{Frm}^2(\mathcal{L})$ of the language $\mathcal{L}$ is defined by adding to ?? the clauses

1. If X is an n -place predicate variable and $t_1, \dots, t_n$ are second-order terms of $\mathcal{L}$, then $X(t_1, \dots, t_n)$ is an atomic *fórmula*.

2. If φ is a fórmula and u is a function variable, then $\forall u \varphi$ is a fórmula.
3. If φ is a fórmula and X is a predicate variable, then $\forall X \varphi$ is a fórmula.
4. If φ is a fórmula and u is a function variable, then $\exists u \varphi$ is a fórmula.
5. If φ is a fórmula and X is a predicate variable, then $\exists X \varphi$ is a fórmula.

Créditos das Imagens

Referências Bibliográficas