-NoValue-

set.1 Comparing Sets

sol:set:cmp: sec

Proposição set.1. *The fórmula* $\forall x\,(X(x) \to Y(x))$ *defines the subset relation, O.s.,* $\mathfrak{M}, s \vDash \forall x\,(X(x) \to Y(x))$ *iff* $s(X) \subseteq s(Y)$.

Proposição set.2. *The fórmula* $\forall x\,(X(x) \leftrightarrow Y(x))$ *defines the identity relation on sets, O.s.,* $\mathfrak{M}, s \vDash \forall x\,(X(x) \leftrightarrow Y(x))$ *iff* $s(X) = s(Y)$.

Proposição set.3. *The fórmula* $\exists x\, X(x)$ *defines the property of being non-empty, O.s.,* $\mathfrak{M}, s \vDash \exists x\, X(x)$ *iff* $s(X) \neq \emptyset$.

A set X is no larger than a set Y, $X \preceq Y$, iff there is uma injetiva function $f\colon X \to Y$. Since we can express that a function is injective, and also that its values for arguments in X are in Y, we can also define the relation of being no larger than on subsets of the domain.

Proposição set.4. *The formula*

$$\exists u\,(\forall x\,(X(x) \to Y(u(x))) \land \forall x\,\forall y\,(u(x) = u(y) \to x = y))$$

defines the relation of being no larger than.

Two sets are the same size, or "equinumerous," $X \approx Y$, iff there is uma bijetiva function $f\colon X \to Y$.

Proposição set.5. *The formula*

$$\begin{aligned}\exists u\,(\forall x\,(X(x) \to Y(u(x))) \land{} \\ \forall x\,\forall y\,(u(x) = u(y) \to x = y) \land{} \\ \forall y\,(Y(y) \to \exists x\,(X(x) \land y = u(x))))\end{aligned}$$

defines the relation of being equinumerous with.

We will abbreviate these fórmulas, respectively, as $X \subseteq Y$, $X = Y$, $X \neq \emptyset$, $X \preceq Y$, and $X \approx Y$. (This may be slightly confusing, since we use the same notation when we speak informally about sets X and Y—but here the notation is an abbreviation for fórmulas in second-order logic involving one-place relation variables X and Y.)

Proposição set.6. *The sentença* $\forall X\,\forall Y\,((X \preceq Y \land Y \preceq X) \to X \approx Y)$ *is valid.*

Prova. The sentença is satisfied in a estrutura $\mathfrak{M}$ if, for any subsets $X \subseteq |\mathfrak{M}|$ and $Y \subseteq |\mathfrak{M}|$, if $X \preceq Y$ and $Y \preceq X$ then $X \approx Y$. But this holds for *any* sets X and Y—it is the Schröder-Bernstein Theorem. □

Créditos das Imagens

Referências Bibliográficas