-NoValue-

syn.1 Semantic Notions

pl:syn:sem:sec

We define the following semantic notions:

Definição syn.1.

1. A fórmula φ is *satisfiable* if for some $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$; it is *unsatisfiable* if for no $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$;
2. A fórmula φ is a *tautology* if $\mathfrak{v} \vDash \varphi$ for all valorações $\mathfrak{v}$;
3. A fórmula φ is *contingent* if it is satisfiable but not a tautology;
4. If Γ is a set of fórmulas, $\Gamma \vDash \varphi$ ("Γ entails φ") if and only if $\mathfrak{v} \vDash \varphi$ for every valoração $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$.
5. If Γ is a set of fórmulas, Γ is *satisfiable* if there is a valoração $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$, and Γ is *unsatisfiable* otherwise.

Problema syn.1. For each of the following four fórmulas determine whether it is (a) satisfiable, (b) tautology, and (c) contingent.

1. $(p_0 \to (\neg p_1 \to \neg p_0))$.
2. $((p_0 \wedge \neg p_1) \to (\neg p_0 \wedge p_2)) \leftrightarrow ((p_2 \to p_0) \to (p_0 \to p_1))$.
3. $(p_0 \leftrightarrow p_1) \to (p_2 \leftrightarrow \neg p_1)$.
4. $((p_0 \leftrightarrow (\neg p_1 \wedge p_2)) \vee (p_2 \to (p_0 \leftrightarrow p_1)))$.

pl:syn:sem:prop:semanticalfacts

Proposição syn.2.

1. *φ is a tautology if and only if $\emptyset \vDash \varphi$;*
2. *If $\Gamma \vDash \varphi$ and $\Gamma \vDash \varphi \to \psi$ then $\Gamma \vDash \psi$;*
3. *If Γ is satisfiable then every finite subset of Γ is also satisfiable;*
4. *Monotonicity: if $\Gamma \subseteq \Delta$ and $\Gamma \vDash \varphi$ then also $\Delta \vDash \varphi$;* pl:syn:sem:def:monotonicity
5. *Transitivity: if $\Gamma \vDash \varphi$ and $\Delta \cup \{\varphi\} \vDash \psi$ then $\Gamma \cup \Delta \vDash \psi$.* pl:syn:sem:def:Cut

Prova. Exercise. □

Problema syn.2. Prove Proposição syn.2

pl:syn:sem:prop:entails-unsat

Proposição syn.3. *$\Gamma \vDash \varphi$ if and only if $\Gamma \cup \{\neg\varphi\}$ is unsatisfiable.*

Prova. Exercise. □

Problema syn.3. Prove Proposição syn.3

Teorema syn.4 (Semantic Deduction Theorem). *$\Gamma \vDash \varphi \to \psi$ if and only if $\Gamma \cup \{\varphi\} \vDash \psi$.* *pl:syn:sem: thm:sem-deduction*

Prova. Exercise. □

Problema syn.4. Prove Teorema syn.4

Créditos das Imagens

Referências Bibliográficas