-NoValue-

syn.1 Preliminaries

pl:syn:pre:sec

pl:syn:pre:thm:induction **Teorema syn.1 (*Principle of induction on fórmulas*).** *If some property P holds for all the atomic fórmulas and is such that*

1. *it holds for $\neg\varphi$ whenever it holds for φ;*
2. *it holds for $(\varphi \land \psi)$ whenever it holds for φ and ψ;*
3. *it holds for $(\varphi \lor \psi)$ whenever it holds for φ and ψ;*
4. *it holds for $(\varphi \to \psi)$ whenever it holds for φ and ψ;*
5. *it holds for $(\varphi \leftrightarrow \psi)$ whenever it holds for φ and ψ;*

then P holds for all fórmulas.

Prova. Let S be the collection of all fórmulas with property P. Clearly $S \subseteq \mathrm{Frm}(\mathcal{L}_0)$. S satisfies all the conditions of **??**: it contains all atomic fórmulas and is closed under the operadores lógicos. $\mathrm{Frm}(\mathcal{L}_0)$ is the smallest such class, so $\mathrm{Frm}(\mathcal{L}_0) \subseteq S$. So $\mathrm{Frm}(\mathcal{L}_0) = S$, and every formula has property P. □

pl:syn:pre:prop:balanced **Proposição syn.2.** *Any fórmula in* $\mathrm{Frm}(\mathcal{L}_0)$ *is* balanced*, in that it has as many left parentheses as right ones.*

Problema syn.1. Prove Proposição syn.2

pl:syn:pre:prop:noinit **Proposição syn.3.** *No proper initial segment of a fórmula is a fórmula.*

Problema syn.2. Prove Proposição syn.3

Proposição syn.4 (Unique Readability). *Any fórmula φ in $\mathrm{Frm}(\mathcal{L}_0)$ has exactly one parsing as one of the following*

1. $\bot$.
2. $\top$.
3. *p_n for some $p_n \in \mathrm{At}_0$.*
4. *$\neg\psi$ for some fórmula ψ.*
5. *$(\psi \land \chi)$ for some fórmulas ψ and χ.*
6. *$(\psi \lor \chi)$ for some fórmulas ψ and χ.*
7. *$(\psi \to \chi)$ for some fórmulas ψ and χ.*

8. *$(\psi \leftrightarrow \chi)$ for some fórmulas ψ and χ.*

Moreover, this parsing is unique*.*

Prova. By induction on φ. For instance, suppose that φ has two distinct readings as $(\psi \to \chi)$ and $(\psi' \to \chi')$. Then ψ and ψ' must be the same (or else one would be a proper initial segment of the other); so if the two readings of φ are distinct it must be because χ and χ' are distinct readings of the same sequence of symbols, which is impossible by the inductive hypothesis. □

Definição syn.5 (Uniform Substitution). If φ and ψ are fórmulas, and p_i is a variável proposicional, then $\varphi[\psi/p_i]$ denotes the result of replacing each occurrence of p_i by an occurrence of ψ in φ; similarly, the simultaneous substitution of $p_1, \ldots, p_n$ by fórmulas $\psi_1, \ldots, \psi_n$ is denoted by $\varphi[\psi_1/p_1, \ldots, \psi_n/p_n]$.

Problema syn.3. For each of the five fórmulas below determine whether the fórmula can be expressed as a substitution $\varphi[\psi/p_i]$ where φ is (i) p_0; (ii) $(\neg p_0 \land p_1)$; and (iii) $((\neg p_0 \to p_1) \land p_2)$. In each case specify the relevant substitution.

1. p_1
2. $(\neg p_0 \land p_0)$
3. $((p_0 \lor p_1) \land p_2)$
4. $\neg((p_0 \to p_1) \land p_2)$
5. $((\neg(p_0 \to p_1) \to (p_0 \lor p_1)) \land \neg(p_0 \land p_1))$

Problema syn.4. Give a mathematically rigorous definition of $\varphi[\psi/p]$ by induction.

Créditos das Imagens

Referências Bibliográficas