-NoValue-

syn.1 Propositional Fórmulas

pl:syn:fml:sec Fórmulas of propositional logic are built up from *variáveis proposicionais*, the propositional constant $\bot$ and the propositional constant $\top$ using *logical connectives*.

1. A infinito enumerável set At_0 of variáveis proposicionais p_0, p_1, ...
2. The propositional constant for falsidade $\bot$.
3. The propositional constant for verdade $\top$.
4. The logical connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (condicional), $\leftrightarrow$ (bicondicional)
5. Punctuation marks: (,), and the comma.

We denote this language of propositional logic by $\mathcal{L}_0$.

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You may be familiar with different terminology and symbols than the ones we use above. Logic texts (and teachers) commonly use either $\sim$, $\neg$, and ! for "negation", $\wedge$, $\cdot$, and & for "conjunction". Commonly used symbols for the "conditional" or "implication" are $\rightarrow$, $\Rightarrow$, and $\supset$. Symbols for "biconditional," "bi-implication," or "(material) equivalence" are $\leftrightarrow$, $\Leftrightarrow$, and $\equiv$. The $\bot$ symbol is variously called "falsity," "falsum," "absurdity," or "bottom." The $\top$ symbol is variously called "truth," "verum," or "top." intro

pl:syn:fml:defn:formulas **Definição syn.1 (Formula).** The set $\mathrm{Frm}(\mathcal{L}_0)$ of *fórmulas* of propositional logic is defined inductively as follows:

1. $\bot$ is an atomic fórmula.
2. $\top$ is an atomic fórmula.
3. Every variável proposicional p_i is an atomic fórmula.
4. If φ is a fórmula, then $\neg\varphi$ is a fórmula.
5. If φ and ψ are fórmulas, then $(\varphi \wedge \psi)$ is a fórmula.
6. If φ and ψ are fórmulas, then $(\varphi \vee \psi)$ is a fórmula.
7. If φ and ψ are fórmulas, then $(\varphi \rightarrow \psi)$ is a fórmula.
8. If φ and ψ are fórmulas, then $(\varphi \leftrightarrow \psi)$ is a fórmula.
9. Nothing else is a fórmula.

explanation

The definition of fórmulas is an *inductive definition.* Essentially, we construct the set of fórmulas in infinitely many stages. In the initial stage, we pronounce all atomic formulas to be formulas; this corresponds to the first few cases of the definition, O.s., the cases for $\top$, $\bot$, p_i. "Atomic fórmula" thus means any fórmula of this form.

The other cases of the definition give rules for constructing new fórmulas out of fórmulas already constructed. At the second stage, we can use them to construct fórmulas out of atomic fórmulas. At the third stage, we construct new formulas from the atomic formulas and those obtained in the second stage, and so on. A fórmula is anything that is eventually constructed at such a stage, and nothing else.

When writing a formula $(\psi * \chi)$ constructed from ψ, χ using a two-place connective $*$, we will often leave out the outermost pair of parentheses and write simply $\psi * \chi$.

Definição syn.2 (Syntactic identity). The symbol $\equiv$ expresses syntactic identity between strings of symbols, O.s., $\varphi \equiv \psi$ iff φ and ψ are strings of symbols of the same length and which contain the same symbol in each place.

The $\equiv$ symbol may be flanked by strings obtained by concatenation, p.ex., $\varphi \equiv (\psi \vee \chi)$ means: the string of symbols φ is the same string as the one obtained by concatenating an opening parenthesis, the string ψ, the $\vee$ symbol, the string χ, and a closing parenthesis, in this order. If this is the case, then we know that the first symbol of φ is an opening parenthesis, φ contains ψ as a substring (starting at the second symbol), that substring is followed by $\vee$, etc.

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Referências Bibliográficas