

-NoValue-

tab.1 Countermodels from Tableaux

nml:tab:cou:
sec The proof of the completeness theorem doesn't just show that if $\models \varphi$ then $\vdash \varphi$, it also gives us a method for constructing countermodels to φ if $\not\models A$. In the case of **K**, this method constitutes a *decision procedure*. For suppose $\not\models \varphi$. Then the proof of ?? gives a method for constructing a complete **tableau**. The method in fact always terminates. The propositional rules for **K** only add prefixed **fórmulas** of lower complexity, O.s., each propositional rule need only be applied once on a branch for any signed formula $\sigma S\varphi$. New prefixes are only generated by the $\Box\mathbb{F}$ and $\Diamond\mathbb{T}$ rules, and also only have to be applied once (and produce a single new prefix). $\Box\mathbb{T}$ and $\Diamond\mathbb{F}$ have to be applied potentially multiple times, but only once per prefix, and only finitely many new prefixes are generated. So the construction either results in a closed branch or a complete branch after finitely many stages. explanation

Once a tableau with an open complete branch is constructed, the proof of ?? gives us an explicit model that satisfies the original set of prefixed **fórmulas**. So not only is it the case that if $\Gamma \models \varphi$, then a closed **tableau** exists and $\Gamma \vdash \varphi$, if we look for the closed **tableau** in the right way and end up with a “complete” **tableau**, we'll not only know that $\Gamma \not\models \varphi$ but actually be able to construct a countermodel.

Exemplo tab.1. We know that $\not\models \Box(p \vee q) \rightarrow (\Box p \vee \Box q)$. The construction of a tableau begins with:

1.	$1\mathbb{F} \Box(p \vee q) \rightarrow (\Box p \vee \Box q) \checkmark$	Assumption
2.	$1\mathbb{T} \Box(p \vee q)$	$\rightarrow\mathbb{F} 1$
3.	$1\mathbb{F} \Box p \vee \Box q \checkmark$	$\rightarrow\mathbb{F} 1$
4.	$1\mathbb{F} \Box p \checkmark$	$\vee\mathbb{F} 3$
5.	$1\mathbb{F} \Box q \checkmark$	$\vee\mathbb{F} 3$
6.	$1.1\mathbb{F} p \checkmark$	$\Box\mathbb{F} 4$
7.	$1.2\mathbb{F} q \checkmark$	$\Box\mathbb{F} 5$

The **tableau** is of course not finished yet. In the next step, we consider the only line without a checkmark: the prefixed **fórmula** $1\mathbb{T}\Box(p \vee q)$ on line 2. The construction of the closed tableau says to apply the $\Box\mathbb{T}$ rule for every prefix used on the branch, O.s., for both 1.1 and 1.2:

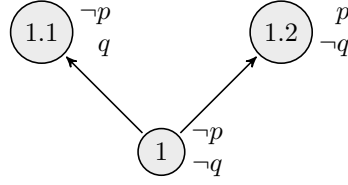


Figura 1: A countermodel to $\Box(p \vee q) \rightarrow (\Box p \vee \Box q)$.

nml:tab:cou:
fig:counter-Box

1.	$1 \text{ F } \Box(p \vee q) \rightarrow (\Box p \vee \Box q) \checkmark$	Assumption
2.	$1 \text{ T } \Box(p \vee q)$	$\rightarrow \text{F } 1$
3.	$1 \text{ F } \Box p \vee \Box q \checkmark$	$\rightarrow \text{F } 1$
4.	$1 \text{ F } \Box p \checkmark$	$\vee \text{F } 3$
5.	$1 \text{ F } \Box q \checkmark$	$\vee \text{F } 3$
6.	$1.1 \text{ F } p \checkmark$	$\Box \text{F } 4$
7.	$1.2 \text{ F } q \checkmark$	$\Box \text{F } 5$
8.	$1.1 \text{ T } p \vee q$	$\Box \text{T } 2$
9.	$1.2 \text{ T } p \vee q$	$\Box \text{T } 2$

Now lines 2, 8, and 9, don't have checkmarks. But no new prefix has been added, so we apply $\vee \text{T}$ to lines 8 and 9, on all resulting branches (as long as they don't close):

1.	$1 \text{ F } \Box(p \vee q) \rightarrow (\Box p \vee \Box q) \checkmark$	Assumption
2.	$1 \text{ T } \Box(p \vee q) \checkmark$	$\rightarrow \text{F } 1$
3.	$1 \text{ F } \Box p \vee \Box q \checkmark$	$\rightarrow \text{F } 1$
4.	$1 \text{ F } \Box p \checkmark$	$\vee \text{F } 3$
5.	$1 \text{ F } \Box q \checkmark$	$\vee \text{F } 3$
6.	$1.1 \text{ F } p \checkmark$	$\Box \text{F } 4$
7.	$1.2 \text{ F } q \checkmark$	$\Box \text{F } 5$
8.	$1.1 \text{ T } p \vee q \checkmark$	$\Box \text{T } 2$
9.	$1.2 \text{ T } p \vee q \checkmark$	$\Box \text{T } 2$
10.	$1.1 \text{ T } p \checkmark$ $\otimes$	$\vee \text{T } 8$
11.	$1.2 \text{ T } p \checkmark$ $\otimes$	$\vee \text{T } 9$

There is one remaining open branch, and it is complete. From it we define the model with worlds $W = \{1, 1.1, 1.2\}$ (the only prefixes appearing on the open branch), the accessibility relation $R = \{\langle 1, 1.1 \rangle, \langle 1, 1.2 \rangle\}$, and the assignment $V(p) = \{1.2\}$ (because line 11 contains $1.2 \text{ T } p$) and $V(q) = \{1.1\}$ (because line 10 contains $1.1 \text{ T } q$). The model is pictured in [Figura 1](#), and you can verify that it is a countermodel to $\Box(p \vee q) \rightarrow (\Box p \vee \Box q)$.

Créditos das Imagens

Referências Bibliográficas