

-NoValue-

seq.1 Sequent Derivações for K

nml:seq:prk:
sec

Exemplo seq.1. We give a sequent calculus *derivação* that shows $\vdash (\Box\varphi \wedge \Box\psi) \rightarrow \Box(\varphi \wedge \psi)$.

$$\begin{array}{c}
\frac{\varphi \Rightarrow \varphi}{\psi, \varphi \Rightarrow \varphi} \quad \frac{\psi \Rightarrow \psi}{\psi, \varphi \Rightarrow \psi} \quad \wedge R \\
\hline
\psi, \varphi \Rightarrow \varphi \wedge \psi \\
\hline
\Box\psi, \Box\varphi \Rightarrow \Box(\varphi \wedge \psi) \quad \Box \\
\hline
\Box\varphi \wedge \Box\psi, \Box\varphi \Rightarrow \Box(\varphi \wedge \psi) \quad \wedge L \\
\hline
\Box\varphi, \Box\varphi \wedge \Box\psi \Rightarrow \Box(\varphi \wedge \psi) \quad XL \\
\hline
\Box\varphi \wedge \Box\psi, \Box\varphi \wedge \Box\psi \Rightarrow \Box(\varphi \wedge \psi) \quad \wedge L \\
\hline
\Box\varphi \wedge \Box\psi \Rightarrow \Box(\varphi \wedge \psi) \quad CL \\
\hline
\Rightarrow (\Box\varphi \wedge \Box\psi) \rightarrow \Box(\varphi \wedge \psi) \quad \rightarrow R
\end{array}$$

Exemplo seq.2. We give a sequent calculus *derivação* that shows $\vdash \Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \vee \Diamond\psi)$.

$$\begin{array}{c}
\frac{\varphi \Rightarrow \varphi}{\varphi \Rightarrow \varphi, \psi} \quad \frac{\psi \Rightarrow \psi}{\psi \Rightarrow \varphi, \psi} \quad \vee L \\
\hline
\varphi \vee \psi \Rightarrow \varphi, \psi \\
\hline
\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi, \Diamond\psi \quad \Diamond \\
\hline
\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi, \Diamond\varphi \vee \Diamond\psi \quad \vee R \\
\hline
\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi \vee \Diamond\psi, \Diamond\varphi \quad XR \\
\hline
\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi \vee \Diamond\psi, \Diamond\varphi \vee \Diamond\psi \quad \vee R \\
\hline
\Diamond(\varphi \vee \psi) \Rightarrow \Diamond\varphi \vee \Diamond\psi \quad CR \\
\hline
\Rightarrow \Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \vee \Diamond\psi) \quad \rightarrow R
\end{array}$$

Here is a *derivação* of DUAL.

$$\begin{array}{c}
\frac{\varphi \Rightarrow \varphi}{\neg\varphi, \varphi \Rightarrow} \neg R \\
\hline
\Diamond\neg\varphi, \Box\varphi \Rightarrow \quad \Diamond \\
\hline
\Box\varphi \Rightarrow \neg\Diamond\neg\varphi \quad \neg R \\
\hline
\Rightarrow \Box\varphi \rightarrow \neg\Diamond\neg\varphi \quad \rightarrow R \\
\hline
\Rightarrow \Box\varphi \leftrightarrow \neg\Diamond\neg\varphi
\end{array}
\quad
\begin{array}{c}
\frac{\varphi \Rightarrow \varphi}{\Rightarrow \varphi, \neg\varphi} \neg R \\
\hline
\Rightarrow \neg\varphi, \varphi \quad XR \\
\hline
\Rightarrow \Diamond\neg\varphi, \Box\varphi \quad \Box \\
\hline
\Rightarrow \Box\varphi, \Diamond\neg\varphi \quad XR \\
\hline
\neg\Diamond\neg\varphi \Rightarrow \Box\varphi \quad \neg R \\
\hline
\Rightarrow \neg\Diamond\neg\varphi \rightarrow \Box\varphi \quad \rightarrow R \\
\hline
\Rightarrow \Box\varphi \leftrightarrow \neg\Diamond\neg\varphi \quad \wedge R
\end{array}$$

Problema seq.1. Find sequent calculus proofs in **K** for the following *fórmulas*:

1. $\Box\neg p \rightarrow \Box(p \rightarrow q)$

2. $(\Box p \vee \Box q) \rightarrow \Box(p \vee q)$

3. $\Diamond p \rightarrow \Diamond(p \vee q)$

4. $\Box(p \wedge q) \rightarrow \Box p$

Créditos das Imagens

Referências Bibliográficas