
$\frac{\varphi, \Gamma \Rightarrow \Delta}{\Box \varphi, \Gamma \Rightarrow \Delta} \text{T}\Box$	$\frac{\Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \Diamond \varphi} \text{T}\Diamond$
$\frac{\Gamma \Rightarrow \Delta}{\Box \Gamma \Rightarrow \Diamond \Delta} \text{D}$	
$\frac{\Gamma, \Diamond \Pi \Rightarrow \Box \Delta, \Lambda, \varphi}{\Box \Gamma, \Pi \Rightarrow \Delta, \Diamond \Lambda, \Box \varphi} \text{B}\Box$	$\frac{\varphi, \Diamond \Gamma, \Pi \Rightarrow \Box \Lambda, \Delta}{\Diamond \varphi, \Gamma, \Box \Pi \Rightarrow \Lambda, \Diamond \Delta} \text{B}\Diamond$
$\frac{\Box \Gamma \Rightarrow \Diamond \Delta, \varphi}{\Box \Gamma \Rightarrow \Diamond \Delta, \Box \varphi} 4\Box$	$\frac{\varphi, \Box \Gamma \Rightarrow \Diamond \Delta}{\Diamond \varphi, \Box \Gamma \Rightarrow \Diamond \Delta} 4\Diamond$
$\frac{\Box \Gamma, \Diamond \Pi \Rightarrow \Box \Delta, \Diamond \Lambda, \varphi}{\Box \Gamma, \Diamond \Pi \Rightarrow \Box \Delta, \Diamond \Lambda, \Box \varphi} 5\Box$	$\frac{\varphi, \Diamond \Gamma, \Box \Pi \Rightarrow \Diamond \Delta, \Box \Lambda}{\Diamond \varphi, \Diamond \Gamma, \Box \Pi \Rightarrow \Diamond \Delta, \Box \Lambda} 5\Diamond$

Tabela 1: More modal rules.

nml:seq;mru:
tab:more-rules

Logic	R is ...	Rules
T = KT	reflexive	$\Box, \text{T}\Box, \text{T}\Diamond$
D = KD	serial	$\Box, \text{D}$
K4	transitive	$\Box, 4\Box, 4\Diamond$
B = KTB	reflexive, symmetric	$\Box, \text{T}\Box, \text{T}\Diamond$ $\text{B}\Box, \text{B}\Diamond$
S4 = KT4	reflexive, transitive	$\Box, \text{T}\Box, \text{T}\Diamond$ $4\Box, 4\Diamond$
S5 = KT5	reflexive, transitive, euclidean	$\Box, \text{T}\Box, \text{T}\Diamond$ $5\Box, 5\Diamond$

Tabela 2: Sequent rules for various modal logics.

nml:seq;mru:
tab:logics-rules

-NoValue-

seq.1 Rules for Other Accessibility Relations

nml:seq;mru:
sec

In order to deal with logics determined by special accessibility relations, we consider the additional rules in [Tabela 1](#).

Adding these rules results in systems that are sound and complete for the logics given in [Tabela 2](#).

Exemplo seq.1. We give a sequent *derivação* that shows **K4** $\vdash$ 4, O.s., $\Box\varphi \rightarrow \Box\Box\varphi$.

$$\frac{\frac{\Box\varphi \Rightarrow \Box\varphi}{\Box\varphi \Rightarrow \Box\Box\varphi} 4\Box}{\Rightarrow \Box\varphi \rightarrow \Box\Box\varphi} \rightarrow R$$

Exemplo seq.2. We give a sequent *derivação* that shows **S5** $\vdash$ 5, O.s., $\Diamond\varphi \rightarrow \Box\Diamond\varphi$.

$$\frac{\frac{\Diamond\varphi \Rightarrow \Diamond\varphi}{\Diamond\varphi \Rightarrow \Box\Diamond\varphi} 5\Box}{\Rightarrow \Diamond\varphi \rightarrow \Box\Diamond\varphi} \rightarrow R$$

Exemplo seq.3. The sequent calculus for **S5** is not complete without the Cut rule; p.ex., $\Diamond\Box\varphi \rightarrow \varphi$, which is valid in **S5**, has no proof without Cut. Here is a *derivação* using Cut:

$$\frac{\frac{\Box\varphi \Rightarrow \Box\varphi}{\Diamond\Box\varphi \Rightarrow \Box\varphi} 5\Diamond \quad \frac{\varphi \Rightarrow \varphi}{\Box\varphi \Rightarrow \varphi} T\Box}{\Diamond\Box\varphi \Rightarrow \varphi} \text{Cut}$$

$$\frac{\Rightarrow \Diamond\Box\varphi \rightarrow \varphi}{\Rightarrow \Diamond\Box\varphi \rightarrow \varphi} \rightarrow R$$

Problema seq.1. Give sequent *derivações* that show the following:

1. **KT5** $\vdash$ B;
2. **KT5** $\vdash$ 4;
3. **KDB4** $\vdash$ T;
4. **KB4** $\vdash$ 5;
5. **KB5** $\vdash$ 4;
6. **KT** $\vdash$ D.

Créditos das Imagens

Referências Bibliográficas