-NoValue-

fil.1 Filtrations are Finite

nml:fil:fin: sec

We've defined filtrations for any set Γ that is closed under sub-fórmulas. Nothing in the definition itself guarantees that filtrations are finite. In fact, when Γ is infinite (p.ex., is the set of all fórmulas), it may well be infinite. However, if Γ is finite (p.ex., when it is the set of sub-fórmulas of a given fórmula φ), so is any filtration through Γ.

nml:fil:fin: prop:filt-are-finite

Proposição fil.1. *If Γ is finite then any filtration $\mathfrak{M}^*$ of a model $\mathfrak{M}$ through Γ is also finite.*

Prova. The size of W^* is the number of different classes $[w]$ under the equivalence relation $\equiv$. Any two worlds u, v in such class—that is, any u and v such that $u \equiv v$—agree on all fórmulas φ in Γ, $\varphi \in \Gamma$ either φ is true at both u and v, or at neither. So each class $[w]$ corresponds to subset of Γ, namely the set of all $\varphi \in \Gamma$ such that φ is true at the worlds in $[w]$. No two different classes $[u]$ and $[v]$ correspond to the same subset of Γ. For if the set of fórmulas true at u and that of fórmulas true at v are the same, then u and v agree on all formulas in Γ, O.s., $u \equiv v$. But then $[u] = [v]$. So, there is uma injetiva function from W^* to $\wp(\Gamma)$, and hence $|W^*| \leq |\wp(\Gamma)|$. Hence if Γ contains n sentences, the cardinality of W^* is no greater than 2^n. □

Créditos das Imagens

Referências Bibliográficas