-NoValue-

fil.1 K and S5 have the Finite Model Property

nml:fil:fmp: sec

Definição fil.1. A system Σ of modal logic is said to have the *finite model property* if whenever a fórmula φ is true at a world in a model of Σ then φ is true at a world in a *finite* model of Σ.

nml:fil:fmp: prop:K-fmp

Proposição fil.2. **K** *has the finite model property.*

Prova. **K** is the set of valid fórmulas, O.s., any model is a model of **K**. By **??**, if $\mathfrak{M}, w \Vdash \varphi$, then $\mathfrak{M}^*, w \Vdash \varphi$ for any filtration of $\mathfrak{M}$ through the set Γ of sub-fórmulas of φ. Any fórmula only has finitely many sub-fórmulas, so Γ is finite. By **??**, $|W^*| \leq 2^n$, where n is the number of fórmulas in Γ. And since **K** imposes no restriction on models, $\mathfrak{M}^*$ is a **K**-model. □

To show that a logic **L** has the finite model property via filtrations it is essential that the filtration of an **L**-model is itself a **L**-model. Often this requires a fair bit of work, and not any filtration yields a **L**-model. However, for universal models, this still holds.

nml:fil:fmp: prop:univ-fin

Proposição fil.3. *Let $\mathcal{U}$ be the class of universal models (see* **??***) and $\mathcal{U}_{\mathrm{Fin}}$ the class of all finite universal models. Then any fórmula φ is valid in $\mathcal{U}$ if and only if it is valid in $\mathcal{U}_{\mathrm{Fin}}$.*

Prova. Finite universal models are universal models, so the left-to-right direction is trivial. For the right-to left direction, suppose that φ is false at some world w in a universal model $\mathfrak{M}$. Let Γ contain φ as well as all of its subformulas; clearly Γ is finite. Take a filtration $\mathfrak{M}^*$ of $\mathfrak{M}$; then $\mathfrak{M}^*$ is finite by **??**, and by **??**, φ is false at $[w]$ in $\mathfrak{M}^*$. It remains to observe that $\mathfrak{M}^*$ is also universal: given u and v, by hypothesis Ruv and by **????**, also $R^*[u][v]$. □

nml:fil:fmp: cor:S5fmp

Corolário fil.4. **S5** *has the finite model property.*

Prova. By **??**, if φ is true at a world in some reflexive and euclidean model then it is true at a world in a universal model. By Proposição fil.3, it is true at a world in a finite universal model (namely the filtration of the model through the set of sub-fórmulas of φ). Every universal model is also reflexive and euclidean; so φ is true at a world in a finite reflexive euclidean model. □

Problema fil.1. Show that any filtration of a serial or reflexive model is also serial or reflexive (respectively).

Problema fil.2. Find a non-symmetric (non-transitive, non-euclidean) filtration of a symmetric (transitive, euclidean) model.

Créditos das Imagens

Referências Bibliográficas

