

-NoValue-

com.1 The Truth Lemma

nml:com:tru:
sec

The canonical model $\mathfrak{M}^\Sigma$ is defined in such a way that $\mathfrak{M}^\Sigma, \Delta \Vdash \varphi$ iff $\varphi \in \Delta$. For **variáveis proposicionais**, the definition of V^Σ yields this directly. We have to verify that the equivalence holds for all **fórmulas**, however. We do this by induction. The inductive step involves proving the equivalence for **fórmulas** involving propositional operators (where we have to use ??) and the modal operators (where we invoke the results of ??).

nml:com:tru:
prop:truthlemma

Proposição com.1 (Truth Lemma). *For every **fórmula** φ , $\mathfrak{M}^\Sigma, \Delta \Vdash \varphi$ if and only if $\varphi \in \Delta$.*

Prova. By induction on φ .

1. $\varphi \equiv \perp$: $\mathfrak{M}^\Sigma, \Delta \nVdash \perp$ by ??, and $\perp \notin \Delta$ by ????
2. $\varphi \equiv \top$: $\mathfrak{M}^\Sigma, \Delta \Vdash \top$ by ??, and $\top \in \Delta$ by ????
3. $\varphi \equiv p$: $\mathfrak{M}^\Sigma, \Delta \Vdash p$ iff $\Delta \in V^\Sigma(p)$ by ??. Also, $\Delta \in V^\Sigma(p)$ iff $p \in \Delta$ by definition of V^Σ .
4. $\varphi \equiv \neg\psi$: $\mathfrak{M}^\Sigma, \Delta \Vdash \neg\psi$ iff $\mathfrak{M}^\Sigma, \Delta \nVdash \psi$ (??) iff $\psi \notin \Delta$ (by inductive hypothesis) iff $\neg\psi \in \Delta$ (by ???).
5. $\varphi \equiv \psi \wedge \chi$: $\mathfrak{M}^\Sigma, \Delta \Vdash \psi \wedge \chi$ iff $\mathfrak{M}^\Sigma, \Delta \Vdash \psi$ and $\mathfrak{M}^\Sigma, \Delta \Vdash \chi$ (by ??) iff $\psi \in \Delta$ and $\chi \in \Delta$ (by inductive hypothesis) iff $\psi \wedge \chi \in \Delta$ (by ???).
6. $\varphi \equiv \psi \vee \chi$: $\mathfrak{M}^\Sigma, \Delta \Vdash \psi \vee \chi$ iff $\mathfrak{M}^\Sigma, \Delta \Vdash \psi$ or $\mathfrak{M}^\Sigma, \Delta \Vdash \chi$ (by ??) iff $\psi \in \Delta$ or $\chi \in \Delta$ (by inductive hypothesis) iff $\psi \vee \chi \in \Delta$ (by ???).
7. $\varphi \equiv \psi \rightarrow \chi$: $\mathfrak{M}^\Sigma, \Delta \Vdash \psi \rightarrow \chi$ iff $\mathfrak{M}^\Sigma, \Delta \nVdash \psi$ or $\mathfrak{M}^\Sigma, \Delta \Vdash \chi$ (by ??) iff $\psi \notin \Delta$ or $\chi \in \Delta$ (by inductive hypothesis) iff $\psi \rightarrow \chi \in \Delta$ (by ???).
8. $\varphi \equiv \psi \leftrightarrow \chi$: $\mathfrak{M}^\Sigma, \Delta \Vdash \psi \leftrightarrow \chi$ iff either $\mathfrak{M}^\Sigma, \Delta \Vdash \psi$ and $\mathfrak{M}^\Sigma, \Delta \Vdash \chi$ or $\mathfrak{M}^\Sigma, \Delta \nVdash \psi$ and $\mathfrak{M}^\Sigma, \Delta \nVdash \chi$ (by ??) iff either $\psi \in \Delta$ and $\chi \in \Delta$ or $\psi \notin \Delta$ and $\chi \notin \Delta$ (by inductive hypothesis) iff $\psi \leftrightarrow \chi \in \Delta$ (by ???).
9. $\varphi \equiv \Box\psi$: First suppose that $\mathfrak{M}^\Sigma, \Delta \Vdash \Box\psi$. By ??, for every Δ' such that $R^\Sigma \Delta \Delta'$, $\mathfrak{M}^\Sigma, \Delta' \Vdash \psi$. By inductive hypothesis, for every Δ' such that $R^\Sigma \Delta \Delta'$, $\psi \in \Delta'$. By definition of R^Σ , for every Δ' such that $\Box^{-1}\Delta \subseteq \Delta'$, $\psi \in \Delta'$. By ??, $\Box\psi \in \Delta$.

Now assume $\Box\psi \in \Delta$. Let $\Delta' \in W^\Sigma$ be such that $R^\Sigma \Delta \Delta'$, O.s., $\Box^{-1}\Delta \subseteq \Delta'$. Since $\Box\psi \in \Delta$, $\psi \in \Box^{-1}\Delta$. Consequently, $\psi \in \Delta'$. By inductive hypothesis, $\mathfrak{M}^\Sigma, \Delta' \Vdash \psi$. Since Δ' is arbitrary with $R^\Sigma \Delta \Delta'$, for all $\Delta' \in W^\Sigma$ such that $R^\Sigma \Delta \Delta'$, $\mathfrak{M}^\Sigma, \Delta' \Vdash \psi$. By ??, $\mathfrak{M}^\Sigma, \Delta \Vdash \Box\psi$.

10. $\varphi \equiv \Diamond\psi$: First suppose that $\mathfrak{M}^\Sigma, \Delta \Vdash \Diamond\psi$. By ??, for some Δ' such that $R^\Sigma \Delta \Delta', \mathfrak{M}^\Sigma, \Delta' \Vdash \psi$. By inductive hypothesis, for some Δ' such that $R^\Sigma \Delta \Delta', \psi \in \Delta'$. By definition of R^Σ , for some Δ' such that $\Box^{-1}\Delta \subseteq \Delta', \psi \in \Delta'$. By ??, for some Δ' such that $\Diamond\Delta' \subseteq \Delta, \psi \in \Delta'$. Since $\psi \in \Delta', \Diamond\psi \in \Diamond\Delta',$ so $\Diamond\psi \in \Delta$.

Now assume $\Diamond\psi \in \Delta$. By ??, there is a complete Σ -consistent $\Delta' \in W^\Sigma$ such that $\Diamond\Delta' \subseteq \Delta$ and $\psi \in \Delta'$. By ??, there is a $\Delta' \in W^\Sigma$ such that $\Box^{-1}\Delta \subseteq \Delta',$ and $\psi \in \Delta'$. By definition of $R^\Sigma, R^\Sigma \Delta \Delta',$ so there is a $\Delta' \in W^\Sigma$ such that $R^\Sigma \Delta \Delta'$ and $\psi \in \Delta'$. By ??, $\mathfrak{M}^\Sigma, \Delta \Vdash \Diamond\psi.$ $\square$

Problema com.1. Complete the proof of **Proposição com.1.**

Créditos das Imagens

Referências Bibliográficas