

-NoValue-

prf.1 Showing Systems are Distinct

In ?? we saw how to prove that two systems of modal logic are in fact the same system. ?? allows us to show that two modal systems Σ and Σ' are distinct, by finding a fórmula φ such that $\Sigma' \vdash \varphi$ that fails in a model of Σ .

Proposição prf.1. $\mathbf{KD} \subsetneq \mathbf{KT}$

Prova. This is the syntactic counterpart to the semantic fact that all reflexive relations are serial. To show $\mathbf{KD} \subseteq \mathbf{KT}$ we need to see that $\mathbf{KD} \vdash \psi$ implies $\mathbf{KT} \vdash \psi$, which follows from $\mathbf{KT} \vdash \mathbf{D}$, as shown in ????. To show that the inclusion is proper, by Soundness (??), it suffices to exhibit a model of $\mathbf{KD}$ where T, O.s., $\Box p \rightarrow p$, fails (an easy task left as an exercise), for then by Soundness $\mathbf{KD} \not\vdash \Box p \rightarrow p$. $\square$

Proposição prf.2. $\mathbf{KB} \neq \mathbf{K4}$.

Prova. We construct a symmetric model where some instance of 4 fails; since obviously the instance is derivável for $\mathbf{K4}$ but not in $\mathbf{KB}$, it will follow $\mathbf{K4} \not\subseteq \mathbf{KB}$. Consider the symmetric model $\mathfrak{M}$ of Figura 1. Since the model is symmetric, K and B are true in $\mathfrak{M}$ (by ?? and ??, respectively). However, $\mathfrak{M}, w_1 \not\models \Box p \rightarrow \Box \Box p$. $\square$

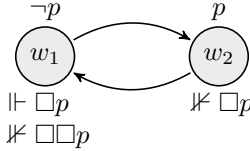


Figura 1: A symmetric model falsifying an instance of 4.

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fig:Bnot4
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thm:KTBnot45

Teorema prf.3. $\mathbf{KTB} \not\vdash 4$ and $\mathbf{KTB} \not\vdash 5$.

Prova. By ?? we know that all instances of T and B are true in every reflexive symmetric model (respectively). So by soundness, it suffices to find a reflexive symmetric model containing a world at which some instance of 4 fails, and similarly for 5. We use the same model for both claims. Consider the symmetric, reflexive model in Figura 2. Then $\mathfrak{M}, w_1 \not\models \Box p \rightarrow \Box \Box p$, so 4 fails at w_1 . Similarly, $\mathfrak{M}, w_2 \not\models \Diamond \neg p \rightarrow \Box \Diamond \neg p$, so the instance of 5 with $\varphi = \neg p$ fails at w_2 . $\square$

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thm:KD5not4

Teorema prf.4. $\mathbf{KD5} \neq \mathbf{KT4} = \mathbf{S4}$.

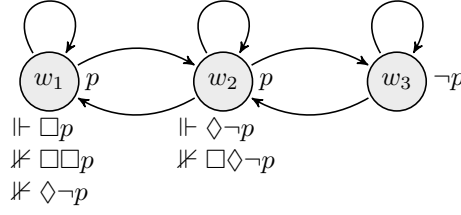


Figura 2: The model for Teorema prf.3.

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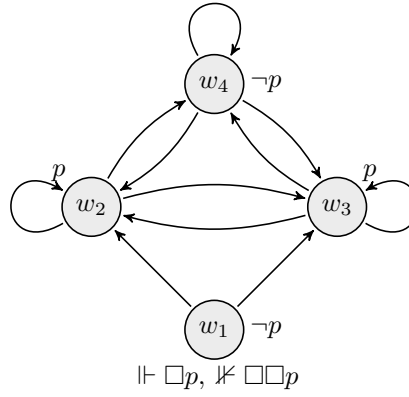


Figura 3: The model for Teorema prf.4.

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Prova. By ?? we know that all instances of D and 5 are true in all serial euclidean models. So it suffices to find a serial euclidean model containing a world at which some instance of 4 fails. Consider the model of Figura 3, and notice that $\mathfrak{M}, w_1 \not\models \Box p \rightarrow \Box \Box p$. $\square$

Problema prf.1. Give an alternative proof of Teorema prf.4 using a model with 3 worlds.

Problema prf.2. Provide a single reflexive transitive model showing that both $\mathbf{KT4} \not\models B$ and $\mathbf{KT4} \not\models 5$.

Créditos das Imagens

Referências Bibliográficas