

-NoValue-

## axs.1 Introduction

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sec

We have a semantics for the basic modal language in terms of modal models, and a notion of a **fórmula** being valid—true at all worlds in all models—or valid with respect to some class of models or frames—true at all worlds in all models in the class, or based on the frame. Logic usually connects such semantic characterizations of validity with a proof-theoretic notion of **derivabilidade**. The aim is to define a notion of **derivabilidade** in some system such that a **fórmula** is **derivável** iff it is valid.

The simplest and historically oldest **derivação** systems are so-called Hilbert-type or axiomatic **derivação** systems. Hilbert-type **derivação** systems for many modal logics are relatively easy to construct: they are simple as objects of metatheoretical study (p.ex., to prove soundness and completeness). However, they are much harder to use to prove **fórmulas** in than, say, natural deduction systems.

In Hilbert-type **derivação** systems, a **derivação** of a **fórmula** is a sequence of **fórmulas** leading from certain axioms, via a handful of inference rules, to the **fórmula** in question. Since we want the **derivação** system to match the semantics, we have to guarantee that the set of **derivável** formulas are true in all models (or true in all models in which all axioms are true). We'll first isolate some properties of modal logics that are necessary for this to work: the “normal” modal logics. For normal modal logics, there are only two inference rules that need to be assumed: modus ponens and necessitation. As axioms we take all (substitution instances) of tautologies, and, depending on the modal logic we deal with, a number of modal axioms. Even if we are just interested in the class of all models, we must also count all substitution instances of K and Dual as axioms. This alone generates the minimal normal modal logic **K**.

**Definição axs.1.** The rule of *modus ponens* is the inference schema

$$\frac{\varphi \quad \varphi \rightarrow \psi}{\psi} \text{ MP}$$

We say a **fórmula**  $\psi$  follows from **fórmulas**  $\varphi, \chi$  by modus ponens iff  $\chi \equiv \varphi \rightarrow \psi$ .

**Definição axs.2.** The rule of *necessitation* is the inference schema

$$\frac{\varphi}{\Box \varphi} \text{ NEC}$$

We say the **fórmula**  $\psi$  follows from the **fórmulas**  $\varphi$  by necessitation iff  $\psi \equiv \Box \varphi$ .

**Definição axs.3.** A **derivação** from a set of axioms  $\Sigma$  is a sequence of **fórmulas**  $\psi_1, \psi_2, \dots, \psi_n$ , where each  $\psi_i$  is either

1. a substitution instance of a tautology, or

2. a substitution instance of a fórmula in  $\Sigma$ , or
3. follows from two fórmulas  $\psi_j, \psi_k$  with  $j, k < i$  by modus ponens, or
4. follows from a fórmula  $\psi_j$  with  $j < i$  by necessitation.

If there is such a derivação with  $\psi_n \equiv \varphi$ , we say that  $\varphi$  is *derivável* from  $\Sigma$ , in symbols  $\Sigma \vdash \varphi$ .

With this definition, it will turn out that the set of *derivável* formulas forms a normal modal logic, and that any *derivável fórmula* is true in every model in which every axiom is true. This property of *derivações* is called *soundness*. The converse, *completeness*, is harder to prove.

## Créditos das Imagens

## Referências Bibliográficas