-NoValue-

bas.1 Reducts and Expansions

Often it is useful or necessary to compare languages which have symbols in common, as well as estruturas for these languages. The most common case is when all the symbols in a linguagem $\mathcal{L}$ are also part of a linguagem $\mathcal{L}'$, O.s., $\mathcal{L} \subseteq \mathcal{L}'$. An $\mathcal{L}$-estrutura $\mathfrak{M}$ can then always be expanded to an $\mathcal{L}'$-estrutura by adding interpretations of the additional symbols while leaving the interpretations of the common symbols the same. On the other hand, from an $\mathcal{L}'$-structure $\mathfrak{M}'$ we can obtain an $\mathcal{L}$-structure simply by "forgetting" the interpretations of the symbols that do not occur in $\mathcal{L}$.

mod:bas:red: defn:reduct **Definição bas.1.** Suppose $\mathcal{L} \subseteq \mathcal{L}'$, $\mathfrak{M}$ is an $\mathcal{L}$-estrutura and $\mathfrak{M}'$ is an $\mathcal{L}'$-estrutura. $\mathfrak{M}$ is the *reduct* of $\mathfrak{M}'$ to $\mathcal{L}$, and $\mathfrak{M}'$ is an *expansion* of $\mathfrak{M}$ to $\mathcal{L}'$ iff

1. $|\mathfrak{M}| = |\mathfrak{M}'|$
2. For every símbolo de constante $c \in \mathcal{L}$, $c^{\mathfrak{M}} = c^{\mathfrak{M}'}$.
3. For every símbolo de função $f \in \mathcal{L}$, $f^{\mathfrak{M}} = f^{\mathfrak{M}'}$.
4. For every símbolo de predicado $P \in \mathcal{L}$, $P^{\mathfrak{M}} = P^{\mathfrak{M}'}$.

mod:bas:red: prop:reduct **Proposição bas.2.** *If an $\mathcal{L}$-estrutura $\mathfrak{M}$ is a reduct of an $\mathcal{L}'$-estrutura $\mathfrak{M}'$, then for all $\mathcal{L}$-sentenças φ,*

$$\mathfrak{M} \vDash \varphi \text{ iff } \mathfrak{M}' \vDash \varphi.$$

Prova. Exercise. □

Problema bas.1. Prove Proposição bas.2.

Definição bas.3. When we have an $\mathcal{L}$-structure $\mathfrak{M}$, and $\mathcal{L}' = \mathcal{L} \cup \{P\}$ is the expansion of $\mathcal{L}$ obtained by adding a single n-place símbolo de predicado P, and $R \subseteq |\mathfrak{M}|^n$ is an n-place relation, then we write $(\mathfrak{M}, R)$ for the expansion $\mathfrak{M}'$ of $\mathfrak{M}$ with $P^{\mathfrak{M}'} = R$.

Créditos das Imagens

Referências Bibliográficas