-NoValue-

bas.1 Overspill

mod:bas:ove: sec

mod:bas:ove: overspill **Teorema bas.1.** *If a set Γ of sentences has arbitrarily large finite models, then it has an infinite model.*

Prova. Expand the language of Γ by adding countably many new constants c_0, c_1, ... and consider the set $\Gamma \cup \{c_i \neq c_j : i \neq j\}$. To say that Γ has arbitrarily large finite models means that for every $m > 0$ there is $n \geq m$ such that Γ has a model of cardinality n. This implies that $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ is finitely satisfiable. By compactness, $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ has a model $\mathfrak{M}$ whose domain must be infinite, since it satisfies all inequalities $c_i \neq c_j$. □

mod:bas:ove: inf-not-fo **Proposição bas.2.** *There is no sentence φ of any first-order language that is true in a estrutura $\mathfrak{M}$ if and only if the domain $|\mathfrak{M}|$ of the estrutura is infinite.*

Prova. If there were such a φ, its negation $\neg\varphi$ would be true in all and only the finite estruturas, and it would therefore have arbitrarily large finite models but it would lack an infinite model, contradicting Teorema bas.1. □

Créditos das Imagens

Referências Bibliográficas