-NoValue-

seq.1 Propositional Rules for Selected Logics

mvl:seq:prl:sec

The inference rules for a connective in an n-sided sequent calculus only depend on the characteristic truth function for the connective. Thus, if some connective is defined by the same truth function in different logics, these n-sided sequent rules for the connective are the same in those logics.

Rules for $\neg$

The following rules for $\neg$ apply to Łukasiewicz and Kleene logics, and their variants.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta}\ \neg\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta}{\Gamma \mid \neg\varphi, \Pi \mid \Delta}\ \neg\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi}\ \neg\mathbb{T}$$

The following rules for $\neg$ apply to Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta}\ \neg_{\mathbf{G}}\mathbb{F} \qquad \frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi}\ \neg_{\mathbf{G}}\mathbb{T}$$

(In Gödel logic, $\neg\varphi$ can never take the value $\mathbb{U}$, so there is no rule for the middle position.)

Rules for $\wedge$

These are the rules for $\wedge$ in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \wedge \psi, \Gamma \mid \Pi \mid \Delta}\ \wedge\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \varphi, \Delta \qquad \Gamma \mid \psi, \Pi \mid \psi, \Delta \qquad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \wedge \psi, \Pi \mid \Delta}\ \wedge\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \qquad \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \wedge \psi} \wedge\mathbb{T}$$

Rules for $\vee$

These are the rules for $\vee$ in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \vee \psi, \Gamma \mid \Pi \mid \Delta} \vee\mathbb{F}$$

$$\frac{\varphi, \Gamma \mid \varphi, \Pi \mid \Delta \qquad \psi, \Gamma \mid \psi, \Pi \mid \Delta \qquad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \vee \psi, \Pi \mid \Delta} \vee\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \vee \psi} \vee\mathbb{T}$$

Rules for $\to$

These are the rules for $\to$ in Łukasiewicz logic.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta} \to_{\mathbf{Ł}_3}\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \psi, \Pi \mid \Delta \qquad \psi, \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta} \to_{\mathbf{Ł}_3}\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \qquad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi} \to_{\mathbf{Ł}_3}\mathbb{T}$$

These are the rules for $\to$ in strong Kleene logic.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta} \to_{\mathbf{Ks}}\mathbb{F}$$

$$\frac{\psi, \Gamma \mid \psi, \Pi \mid \Delta \qquad \Gamma \mid \varphi, \psi, \Pi \mid \Delta \qquad \Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta} \to_{\mathbf{Ks}}\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi} \to_{\mathbf{Ks}} \mathbb{T}$$

These are the rules for $\to$ in Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta} \to_{\mathbf{G}_3} \mathbb{F}$$

$$\frac{\Gamma \mid \psi, \Pi \mid \Delta \qquad \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta} \to_{\mathbf{G}_3} \mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \qquad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi} \to_{\mathbf{G}_3} \mathbb{T}$$

Créditos das Imagens

Referências Bibliográficas

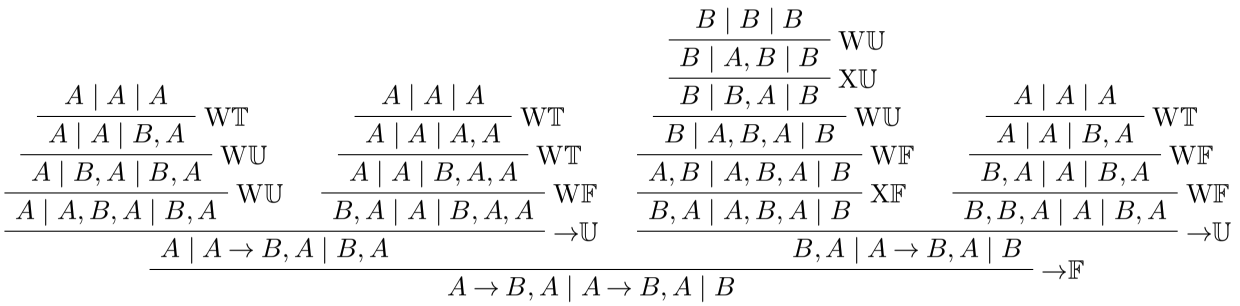

Figura 1: Example derivação in $\mathbf{Ł}_3$