

-NoValue-

seq.1 Propositional Rules for Selected Logics

mv1:seq:prl:
sec

The inference rules for a connective in an n -sided sequent calculus only depend on the characteristic truth function for the connective. Thus, if some connective is defined by the same truth function in different logics, these n -sided sequent rules for the connective are the same in those logics.

Rules for $\neg$

The following rules for $\neg$ apply to Łukasiewicz and Kleene logics, and their variants.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta} \neg\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta}{\Gamma \mid \neg\varphi, \Pi \mid \Delta} \neg\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi} \neg\mathbb{T}$$

The following rules for $\neg$ apply to Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta} \neg\mathbf{G}\mathbb{F} \qquad \frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi} \neg\mathbf{G}\mathbb{T}$$

(In Gödel logic, $\neg\varphi$ can never take the value $\mathbb{U}$, so there is no rule for the middle position.)

Rules for $\wedge$

These are the rules for $\wedge$ in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \wedge \psi, \Gamma \mid \Pi \mid \Delta} \wedge\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \varphi, \Delta \quad \Gamma \mid \psi, \Pi \mid \psi, \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \wedge \psi, \Pi \mid \Delta} \wedge\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \wedge \psi} \wedge \mathbb{T}$$

Rules for $\vee$

These are the rules for $\vee$ in Łukasiewicz, strong Kleene, and Gödel logic.

$$\begin{array}{c} \frac{\varphi, \Gamma \mid \Pi \mid \Delta \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \vee \psi, \Gamma \mid \Pi \mid \Delta} \vee \mathbb{F} \\[10pt] \frac{\varphi, \Gamma \mid \varphi, \Pi \mid \Delta \quad \psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \vee \psi, \Pi \mid \Delta} \vee \mathbb{U} \\[10pt] \frac{\Gamma \mid \Pi \mid \Delta, \varphi, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \vee \psi} \vee \mathbb{T} \end{array}$$

Rules for $\rightarrow$

These are the rules for $\rightarrow$ in Łukasiewicz logic.

$$\begin{array}{c} \frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \rightarrow \psi, \Gamma \mid \Pi \mid \Delta} \rightarrow_{\mathbf{L}_3} \mathbb{F} \\[10pt] \frac{\Gamma \mid \varphi, \psi, \Pi \mid \Delta \quad \psi, \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \rightarrow \psi, \Pi \mid \Delta} \rightarrow_{\mathbf{L}_3} \mathbb{U} \\[10pt] \frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \quad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \rightarrow \psi} \rightarrow_{\mathbf{L}_3} \mathbb{T} \end{array}$$

These are the rules for $\rightarrow$ in strong Kleene logic.

$$\begin{array}{c} \frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \rightarrow \psi, \Gamma \mid \Pi \mid \Delta} \rightarrow_{\mathbf{Ks}} \mathbb{F} \\[10pt] \frac{\psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \rightarrow \psi, \Pi \mid \Delta} \rightarrow_{\mathbf{Ks}} \mathbb{U} \end{array}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \rightarrow \psi} \rightarrow_{\mathbf{Ks}} \mathbb{T}$$

These are the rules for $\rightarrow$ in Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \rightarrow \psi, \Gamma \mid \Pi \mid \Delta} \rightarrow_{\mathbf{G}_3} \mathbb{F}$$

$$\frac{\Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \rightarrow \psi, \Pi \mid \Delta} \rightarrow_{\mathbf{G}_3} \mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \quad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \rightarrow \psi} \rightarrow_{\mathbf{G}_3} \mathbb{T}$$

Créditos das Imagens

Referências Bibliográficas

$$\begin{array}{c}
\frac{B \mid B \mid B}{B \mid A, B \mid B} \text{WU} \\
\frac{B \mid B, A \mid B}{B \mid A, B \mid B} \text{XU} \\
\frac{B \mid A, B, A \mid B}{A, B \mid A, B, A \mid B} \text{WU} \\
\frac{B, A \mid A, B, A \mid B}{B, A \mid A, B, A \mid B} \text{XFF} \\
\frac{A \mid A \mid A}{A \mid A \mid B, A} \text{WTT} \\
\frac{B, A \mid A \mid B, A}{B, B, A \mid A \mid B, A} \text{WFF} \\
\frac{A \mid A \mid A}{A \mid A \mid B, A} \text{WTT} \\
\frac{A \mid A \mid B, A}{A \mid B, A \mid B, A} \text{WU} \\
\frac{A \mid A, B, A \mid B, A}{A \mid A \rightarrow B, A \mid B, A} \text{WU} \\
\frac{A \mid A \rightarrow B, A \mid B, A}{A \rightarrow B, A \mid A \rightarrow B, A \mid B} \rightarrow \text{U}
\end{array}$$

Figura 1: Example derivação in $\mathbf{L}_3$