-NoValue-

cr.1 Parallel $\beta\eta$-reduction

lam:cr:pbe: sec

In this section we prove the Church-Rosser property for parallel $\beta\eta$-reduction, the parallel reduction notion corresponding to $\beta\eta$-reduction.

lam:cr:pbe: defn:beredpar

Definição cr.1 (Parallel $\beta\eta$-reduction, $\overset{\beta\eta}{\Longrightarrow}$). *Parallel $\beta\eta$-reduction* ($\overset{\beta\eta}{\Longrightarrow}$) on terms is inductively defined as follows:

1. lam:cr:pbe: defn:beredpar1 — $x \overset{\beta\eta}{\Longrightarrow} x$.
2. lam:cr:pbe: defn:beredpar2 — If $N \xrightarrow{\beta} N'$ then $\lambda x.\, N \overset{\beta\eta}{\Longrightarrow} \lambda x.\, N'$.
3. lam:cr:pbe: defn:beredpar3 — If $P \overset{\beta\eta}{\Longrightarrow} P'$ and $Q \overset{\beta\eta}{\Longrightarrow} Q'$ then $PQ \overset{\beta\eta}{\Longrightarrow} P'Q'$.
4. lam:cr:pbe: defn:beredpar4 — If $N \overset{\beta\eta}{\Longrightarrow} N'$ and $Q \overset{\beta\eta}{\Longrightarrow} Q'$ then $(\lambda x.\, N)Q \overset{\beta\eta}{\Longrightarrow} N'[Q'/x]$.
5. lam:cr:pbe: defn:beredpar5 — If $N \overset{\beta\eta}{\Longrightarrow} N'$ then $\lambda x.\, Nx \overset{\beta\eta}{\Longrightarrow} N'$, provided $x \notin FV(N)$.

lam:cr:pbe: thm:refl

Teorema cr.2. *$M \overset{\beta\eta}{\Longrightarrow} M$.*

Prova. Exercise. □

Problema cr.1. Prove Teorema cr.2.

lam:cr:pbe: defn:becd

Definição cr.3 ($\beta\eta$-complete development). The *$\beta\eta$-complete development* $M^{*\beta\eta}$ of M is defined as follows:

lam:cr:pbe: defn:becd1, lam:cr:pbe: defn:becd2, lam:cr:pbe: defn:becd3, lam:cr:pbe: defn:becd4, lam:cr:pbe: defn:becd5

$$
\begin{aligned}
x^{*\beta\eta} &= x && (1)\\
(\lambda x.\, N)^{*\beta\eta} &= \lambda x.\, N^{*\beta\eta} && (2)\\
(PQ)^{*\beta\eta} &= P^{*\beta\eta}Q^{*\beta\eta} && \text{if } P \text{ is not a } \lambda\text{-abstract} \quad (3)\\
((\lambda x.\, N)Q)^{*\beta\eta} &= N^{*\beta\eta}[Q^{*\beta\eta}/x] && (4)\\
(\lambda x.\, Nx)^{*\beta\eta} &= N^{*\beta\eta} \qquad \text{if } x \notin FV(N) && (5)
\end{aligned}
$$

lam:cr:pbe: lem:comp

Lema cr.4. *If $M \overset{\beta\eta}{\Longrightarrow} M'$ and $R \overset{\beta\eta}{\Longrightarrow} R'$, then $M[R/y] \overset{\beta\eta}{\Longrightarrow} M'[R'/y]$.*

Prova. By induction on the derivação of $M \overset{\beta\eta}{\Longrightarrow} M'$.

The first four cases are exactly like those in **??**. If the last rule is (5), then M is $\lambda x.\, Nx$, M' is N' for some x and N' where $x \notin FV(N)$, and $N \overset{\beta\eta}{\Longrightarrow} N'$. We want to show that $(\lambda x.\, Nx)[R/y] \overset{\beta\eta}{\Longrightarrow} N'[R'/y]$, O.s., $\lambda x.\, N[R/y]x \overset{\beta\eta}{\Longrightarrow} N'[R'/y]$. It follows by Definição cr.1(5) and the induction hypothesis. □

Lema cr.5. *If* $M \overset{\beta\eta}{\Longrightarrow} M'$ *then* $M' \overset{\beta\eta}{\Longrightarrow} M^{*\beta\eta}$. *lam:cr:pbe:lem:cont*

Prova. By induction on the derivação of $M \overset{\beta\eta}{\Longrightarrow} M'$.

The first four cases are like those in **??**. If the last rule is (5), then M is $\lambda x.\, Nx$ and M' is N' for some x, N, N' where $x \notin FV(N)$ and $N \overset{\beta\eta}{\Longrightarrow} N'$. We want to show that $N' \overset{\beta\eta}{\Longrightarrow} (\lambda x.\, Nx)^{*\beta\eta}$, O.s., $N' \overset{\beta\eta}{\Longrightarrow} N^{*\beta\eta}$, which is immediate by induction hypothesis. □

Teorema cr.6. $\overset{\beta\eta}{\Longrightarrow}$ *has the Church-Rosser property.* *lam:cr:pbe:thm:cr*

Prova. Immediate from Lema cr.5. □

Créditos das Imagens

Referências Bibliográficas