

-NoValue-

sc.1 Decidability

int:sc:dec:
sec Observe that the proof of the completeness theorem gives us for every $\Gamma \not\models \varphi$ a model with an infinite number of worlds witnessing the fact that $\Gamma \not\models \varphi$. The following proposition shows that to prove $\models \varphi$ it is enough to prove that $\mathfrak{M} \models \varphi$ for all finite models (O.s., models with a finite set of worlds).

int:sc:dec:
thm:decidability **Teorema sc.1.** *If $\not\models \varphi$ then there is a finite model $\mathfrak{M}' \not\models \varphi$.*

Prova. Assume $\mathfrak{M} = \langle W, R, V \rangle$ is such that $\mathfrak{M} \not\models \varphi$ and P is the set of **variáveis proposicionais** occurring in φ . Define $\mathfrak{M}' = \langle W', R', V' \rangle$ by letting $W' = \{[w] : w \in W\}$ where $[w] = \{p \in P : w \in V(p)\}$, R' be the subset relation, and $V'(p) = \{[w] : p \in [w]\}$. It should be clear that W' is a finite set and that $\mathfrak{M}'$ is a **modelo relacional**.

It can be shown, by induction on φ , that

$$\mathfrak{M}, w \models \varphi \text{ iff } \mathfrak{M}', [w] \models \varphi$$

for all **fórmulas** φ with only **variáveis proposicionais** from P . This is left as an exercise for the reader. $\square$

Problema sc.1. Finish the proof of **Teorema sc.1** by showing that $\mathfrak{M}, w \models \varphi$ iff $\mathfrak{M}', [w] \models \varphi$ for all **fórmulas** φ with only propositional variables from P .

From **Teorema sc.1** it follows that there is an algorithm to decide whether $\models \varphi$.

Créditos das Imagens

Referências Bibliográficas