

-NoValue-

## sc.1 The Completeness Theorem

int:sc:cpl:

sec

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thm:completeness

**Teorema sc.1.** *If  $\Gamma \models \varphi$  then  $\Gamma \vdash \varphi$ .*

*Prova.* We prove the contrapositive: Suppose  $\Gamma \not\models \varphi$ . Then by ??, there is a prime set  $\Gamma^* \supseteq \Gamma$  such that  $\Gamma^* \not\models \varphi$ . Consider the canonical model  $\mathfrak{M}(\Gamma^*)$  for  $\Gamma^*$  as defined in ??. For any  $\psi \in \Gamma$ ,  $\Gamma^* \vdash \psi$ . Note that  $\Gamma^*(A) = \Gamma^*$ . By the Truth Lemma (??), we have  $\mathfrak{M}(\Gamma^*), A \models \psi$  for all  $\psi \in \Gamma$  and  $\mathfrak{M}(\Gamma^*), A \not\models \varphi$ . This shows that  $\Gamma \not\models \varphi$ .  $\square$

**Problema sc.1.** Show that if  $\varphi$  only contains **variáveis proposicionais**,  $\vee$ , and  $\wedge$ , then  $\not\models \varphi$ . Use this to conclude that  $\rightarrow$  is not definable in intuitionistic logic from  $\vee$  and  $\wedge$ .

**Problema sc.2.** By using the completeness theorem prove that if  $\vdash \varphi \vee \psi$  then  $\vdash \varphi$  or  $\vdash \psi$ . (Hint: Assume  $\mathfrak{M}_1 \not\models \varphi$  and  $\mathfrak{M}_2 \not\models \psi$  and construct a new model  $\mathfrak{M}$  such that  $\mathfrak{M} \not\models \varphi \vee \psi$ .)

**Problema sc.3.** Show that if  $\mathfrak{M}$  is a relational model using a linear order then  $\mathfrak{M} \models (\varphi \rightarrow \psi) \vee (\psi \rightarrow \varphi)$ .

## Créditos das Imagens

## Referências Bibliográficas