

-NoValue-

req.1 Representing Relations

inc:req:rel:
sec Let us say what it means for a *relation* to be representable.

inc:req:rel:
defn:representing-relations **Definição req.1.** A relation $R(x_0, \dots, x_k)$ on the natural numbers is *representable in $\mathbf{Q}$* if there is a formula $\varphi_R(x_0, \dots, x_k)$ such that whenever $R(n_0, \dots, n_k)$ is true, $\mathbf{Q}$ proves $\varphi_R(\bar{n}_0, \dots, \bar{n}_k)$, and whenever $R(n_0, \dots, n_k)$ is false, $\mathbf{Q}$ proves $\neg\varphi_R(\bar{n}_0, \dots, \bar{n}_k)$.

inc:req:rel:
thm:representing-rels **Teorema req.2.** A relation is representable in $\mathbf{Q}$ if and only if it is computable.

Prova. For the forwards direction, suppose $R(x_0, \dots, x_k)$ is represented by the formula $\varphi_R(x_0, \dots, x_k)$. Here is an algorithm for computing R : on input $n_0, \dots, n_k$, simultaneously search for a proof of $\varphi_R(\bar{n}_0, \dots, \bar{n}_k)$ and a proof of $\neg\varphi_R(\bar{n}_0, \dots, \bar{n}_k)$. By our hypothesis, the search is bound to find one or the other; if it is the first, report “yes,” and otherwise, report “no.”

In the other direction, suppose $R(x_0, \dots, x_k)$ is computable. By definition, this means that the function $\chi_R(x_0, \dots, x_k)$ is computable. By ??, χ_R is represented by a formula, say $\varphi_{\chi_R}(x_0, \dots, x_k, y)$. Let $\varphi_R(x_0, \dots, x_k)$ be the formula $\varphi_{\chi_R}(x_0, \dots, x_k, \bar{1})$. Then for any $n_0, \dots, n_k$, if $R(n_0, \dots, n_k)$ is true, then $\chi_R(n_0, \dots, n_k) = 1$, in which case $\mathbf{Q}$ proves $\varphi_{\chi_R}(\bar{n}_0, \dots, \bar{n}_k, \bar{1})$, and so $\mathbf{Q}$ proves $\varphi_R(\bar{n}_0, \dots, \bar{n}_k)$. On the other hand, if $R(n_0, \dots, n_k)$ is false, then $\chi_R(n_0, \dots, n_k) = 0$. This means that $\mathbf{Q}$ proves

$$\forall y (\varphi_{\chi_R}(\bar{n}_0, \dots, \bar{n}_k, y) \rightarrow y = \bar{0}).$$

Since $\mathbf{Q}$ proves $\bar{0} \neq \bar{1}$, $\mathbf{Q}$ proves $\neg\varphi_{\chi_R}(\bar{n}_0, \dots, \bar{n}_k, \bar{1})$, and so it proves $\neg\varphi_R(\bar{n}_0, \dots, \bar{n}_k)$. $\square$

Problema req.1. Show that if R is representable in $\mathbf{Q}$, so is χ_R .

Créditos das Imagens

Referências Bibliográficas