-NoValue-

art.1 Coding Symbols

inc:art:cod:sec The basic language $\mathcal{L}$ of first order logic makes use of the symbols

$$\bot \quad \neg \quad \lor \quad \land \quad \to \quad \forall \quad \exists \quad = \quad (\quad) \quad ,$$

together with enumerável sets of variables and símbolos de constante, and enumerável sets of símbolos de função and símbolos de predicado of arbitrary arity. We can assign *codes* to each of these symbols in such a way that every symbol is assigned a unique number as its code, and no two different symbols are assigned the same number. We know that this is possible since the set of all symbols is enumerável and so there is uma bijeção between it and the set of natural numbers. But we want to make sure that we can recover the symbol (as well as some information about it, p.ex., the arity of a símbolo de função) from its code in a computable way. There are many possible ways of doing this, of course. Here is one such way, which uses primitive recursive functions. (Recall that $\langle n_0, \dots, n_k \rangle$ is the number coding the sequence of numbers n_0, $\dots$, n_k.)

Definição art.1. If s is a symbol of $\mathcal{L}$, let the *symbol code* c_s be defined as follows:

1. If s is among the logical symbols, c_s is given by the following table:

$\bot$	$\neg$	$\lor$	$\land$	$\to$	$\forall$
$\langle 0,0\rangle$	$\langle 0,1\rangle$	$\langle 0,2\rangle$	$\langle 0,3\rangle$	$\langle 0,4\rangle$	$\langle 0,5\rangle$
$\exists$	$=$	(	)	,	
$\langle 0,6\rangle$	$\langle 0,7\rangle$	$\langle 0,8\rangle$	$\langle 0,9\rangle$	$\langle 0,10\rangle$	

2. If s is the i-th variable v_i, then $\mathrm{c}_s = \langle 1, i\rangle$.
3. If s is the i-th símbolo de constante c_i, then $\mathrm{c}_s = \langle 2, i\rangle$.
4. If s is the i-th n-ary símbolo de função f_i^n, then $\mathrm{c}_s = \langle 3, n, i\rangle$.
5. If s is the i-th n-ary símbolo de predicado P_i^n, then $\mathrm{c}_s = \langle 4, n, i\rangle$.

Proposição art.2. *The following relations are primitive recursive:*

1. $\mathrm{Fn}(x, n)$ *iff x is the code of f_i^n for some i, O.s., x is the code of an n-ary símbolo de função.*
2. $\mathrm{Pred}(x, n)$ *iff x is the code of P_i^n for some i or x is the code of $=$ and $n = 2$, O.s., x is the code of an n-ary símbolo de predicado.*

Definição art.3. If $s_0, \dots, s_{n-1}$ is a sequence of symbols, its *Gödel number* is $\langle \mathrm{c}_{s_0}, \dots, \mathrm{c}_{s_{n-1}}\rangle$.

explanation

Note that *codes* and *Gödel numbers* are different things. For instance, the variable v_5 has a code $\mathrm{c}_{\mathsf{v}_5} = \langle 1, 5\rangle = 2^2 \cdot 3^6$. But the variable v_5 considered as a term is also a sequence of symbols (of length 1). The *Gödel number* ${}^{\#}\mathsf{v}_5{}^{\#}$ of the *term* v_5 is $\langle \mathrm{c}_{\mathsf{v}_5}\rangle = 2^{\mathrm{c}_{\mathsf{v}_5}+1} = 2^{2^2\cdot 3^6+1}$.

Exemplo art.4. Recall that if $k_0, \ldots, k_{n-1}$ is a sequence of numbers, then the code of the sequence $\langle k_0, \ldots, k_{n-1}\rangle$ in the power-of-primes coding is

$$2^{k_0+1} \cdot 3^{k_1+1} \cdot \cdots \cdot p_{n-1}^{k_{n-1}+1},$$

where p_i is the i-th prime (starting with $p_0 = 2$). So for instance, the formula $\mathsf{v}_0 = \mathsf{o}$, or, more explicitly, $=(\mathsf{v}_0, \mathsf{c}_0)$, has the Gödel number

$$\langle \mathrm{c}_=, \mathrm{c}_(, \mathrm{c}_{\mathsf{v}_0}, \mathrm{c}_,, \mathrm{c}_{\mathsf{c}_0}, \mathrm{c}_)\rangle.$$

Here, $\mathrm{c}_=$ is $\langle 0, 7\rangle = 2^{0+1} \cdot 3^{7+1}$, $\mathrm{c}_{\mathsf{v}_0}$ is $\langle 1, 0\rangle = 2^{1+1} \cdot 3^{0+1}$, etc. So ${}^{\#}{=}(\mathsf{v}_0, \mathsf{c}_0)^{\#}$ is

$$\begin{aligned}2^{\mathrm{c}_=+1} \cdot 3^{\mathrm{c}_(+1} \cdot 5^{\mathrm{c}_{\mathsf{v}_0}+1} \cdot 7^{\mathrm{c}_,+1} \cdot 11^{\mathrm{c}_{\mathsf{c}_0}+1} \cdot 13^{\mathrm{c}_)+1} = \\ 2^{2^1\cdot 3^8+1} \cdot 3^{2^1\cdot 3^9+1} \cdot 5^{2^2\cdot 3^1+1} \cdot 7^{2^1\cdot 3^{11}+1} \cdot 11^{2^3\cdot 3^1+1} \cdot 13^{2^1\cdot 3^{10}+1} = \\ 2^{13\,123} \cdot 3^{39\,367} \cdot 5^{13} \cdot 7^{354\,295} \cdot 11^{25} \cdot 13^{118\,099}.\end{aligned}$$

Créditos das Imagens

Referências Bibliográficas