-NoValue-

art.1 Coding Fórmulas

inc:art:frm:sec Once we have defined the relation $\mathrm{Term}(x)$ primitive recursively, we can use it to define the corresponding relation for fórmulas, $\mathrm{Frm}(x)$ primitive recursively.

Proposição art.1. *The relation* $\mathrm{Atom}(x)$ *which holds iff* x *is the Gödel number of an atomic fórmula, is primitive recursive.*

Prova. The number x is the Gödel number of an atomic fórmula iff one of the following holds:

1. There are n, $j < x$, and $z < x$ such that for each $i < n$, $\mathrm{Term}((z)_i)$ and $x =$
$$\ulcorner P^n_j(\urcorner \frown \mathrm{flatten}(z) \frown \ulcorner)\urcorner.$$
2. There are $z_1, z_2 < x$ such that $\mathrm{Term}(z_1)$, $\mathrm{Term}(z_2)$, and $x =$
$$\ulcorner{=}(\urcorner \frown z_1 \frown \ulcorner,\urcorner \frown z_2 \frown \ulcorner)\urcorner.$$
3. $x = \ulcorner\bot\urcorner$.
4. $x = \ulcorner\top\urcorner$. □

inc:art:frm:prop:frm-primrec **Proposição art.2.** *The relation* $\mathrm{Frm}(x)$ *which holds iff* x *is the Gödel number of a fórmula is primitive recursive.*

Prova. A sequence of symbols s is a fórmula iff there is formation sequence s_0, …, $s_{k-1} = s$ of fórmula which records how s was formed from atomic fórmulas according to the formation rules. The code for each s_i (and indeed of the code of the sequence $\langle s_0, \dots, s_{k-1}\rangle$) is less than the code x of s. □

Problema art.1. Give a detailed proof of Proposição art.2 along the lines of the first proof of **??**.

inc:art:frm:prop:freeocc-primrec **Proposição art.3.** *The relation* $\mathrm{FreeOcc}(x, z, i)$, *which holds iff the* i*-th symbol of the formula with Gödel number* x *is a free occurrence of the variable with Gödel number* z*, is primitive recursive.*

Prova. Exercise. □

Problema art.2. Prove Proposição art.3. You may make use of the fact that any substring of a fórmula which is a fórmula is a sub-fórmula of it.

Proposição art.4. *The property* $\mathrm{Sent}(x)$ *which holds iff* x *is the Gödel number of a sentença is primitive recursive.*

Prova. A sentença is a fórmula without free occurrences of variáveis. So $\mathrm{Sent}(x)$ holds iff

$$(\forall i < \mathrm{len}(x))\ (\forall z < x) \quad ((\exists j < z)\ z = {}^{\#}\mathsf{v}_j{}^{\#} \to \neg \mathrm{FreeOcc}(x, z, i)). \qquad \square$$

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Referências Bibliográficas