

-NoValue-

tab.1 Derivabilidade and the Quantifiers

fol:tab:qpr:sec The completeness theorem also requires that the tableaux rules yield the facts explanation about $\vdash$ established in this section.

fol:tab:qpr:thm:strong-generalization **Teorema tab.1.** *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

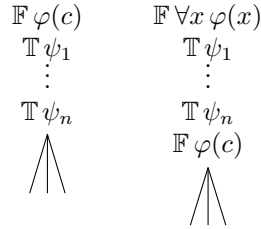
Prova. Suppose $\Gamma \vdash \varphi(c)$, O.s., there are $\psi_1, \dots, \psi_n \in \Gamma$ and a closed **tableau** for

$$\{\mathbb{F} \varphi(c), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

We have to show that there is also a closed **tableau** for

$$\{\mathbb{F} \forall x \varphi(x), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

Take the closed **tableau** and replace the first assumption with $\mathbb{F} \forall x \varphi(x)$, and insert $\mathbb{F} \varphi(c)$ after the assumptions.



The tableau is still closed, since all **sentenças** available as assumptions before are still available at the top of the **tableau**. The inserted line is the result of a correct application of $\forall\mathbb{F}$, since the **símbolo de constante** c does not occur in $\psi_1, \dots, \psi_n$ or $\forall x \varphi(x)$, O.s., it does not occur above the inserted line in the new **tableau**. $\square$

fol:tab:qpr:prop:provability-quantifiers

Proposição tab.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

Prova. 1. A closed **tableau** for $\mathbb{F} \exists x \varphi(x), \mathbb{T} \varphi(t)$ is:

- | | | |
|-----------|-----------------------------------|-----------------------|
| 1. | $\mathbb{F} \exists x \varphi(x)$ | Assumption |
| 2. | $\mathbb{T} \varphi(t)$ | Assumption |
| 3. | $\mathbb{F} \varphi(t)$ | $\exists\mathbb{F} 1$ |
| $\otimes$ | | |

2. A closed **tableau** for $\mathbb{F} \varphi(t), \mathbb{T} \forall x \varphi(x)$, is:

- | | | |
|----|-----------------------------------|------------------------|
| 1. | $\mathbb{F} \varphi(t)$ | Assumption |
| 2. | $\mathbb{T} \forall x \varphi(x)$ | Assumption |
| 3. | $\mathbb{T} \varphi(t)$ | $\forall \mathbb{T} 2$ |
| | $\otimes$ | |

Créditos das Imagens

Referências Bibliográficas