

-NoValue-

rec.1 Primitive Recursion Notations

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One advantage to having the precise inductive description of the primitive recursive functions is that we can be systematic in describing them. For example, we can assign a “notation” to each such function, as follows. Use symbols zero, succ, and P_i^n for zero, successor, and the projections. Now suppose h is defined by composition from a k -place function f and n -place functions $g_0, \dots, g_{k-1}$, and we have assigned notations $F, G_0, \dots, G_{k-1}$ to the latter functions. Then, using a new symbol $\text{Comp}_{k,n}$, we can denote the function h by $\text{Comp}_{k,n}[F, G_0, \dots, G_{k-1}]$.

For functions defined by primitive recursion, we can use analogous notations. Suppose the $(k+1)$ -ary function h is defined by primitive recursion from the k -ary function f and the $(k+2)$ -ary function g , and the notations assigned to f and g are F and G , respectively. Then the notation assigned to h is $\text{Rec}_k[F, G]$.

Recall that the addition function is defined by primitive recursion as

$$\begin{aligned}\text{add}(x_0, 0) &= P_0^1(x_0) = x_0 \\ \text{add}(x_0, y + 1) &= \text{succ}(P_2^3(x_0, y, \text{add}(x_0, y))) = \text{add}(x_0, y) + 1\end{aligned}$$

Here the role of f is played by P_0^1 , and the role of g is played by $\text{succ}(P_2^3(x_0, y, z))$, which is assigned the notation $\text{Comp}_{1,3}[\text{succ}, P_2^3]$ as it is the result of defining a function by composition from the 1-ary function succ and the 3-ary function P_2^3 . With this setup, we can denote the addition function by

$$\text{Rec}_1[P_0^1, \text{Comp}_{1,3}[\text{succ}, P_2^3]].$$

Having these notations sometimes proves useful, p.ex., when enumerating primitive recursive functions.

Problema rec.1. Give the complete primitive recursive notation for mult.

Créditos das Imagens

Referências Bibliográficas