

-NoValue-

thy.1 There Are Non-Computable Sets

cmp:thy:nep:
sec We saw above that every computable set is computably enumerable. Is the converse true? The following shows that, in general, it is not.

cmp:thy:nep:
thm:K-0 **Teorema thy.1.** *Let K_0 be the set $\{\langle e, x \rangle : \varphi_e(x) \downarrow\}$. Then K_0 is computably enumerable but not computable.*

Prova. To see that K_0 is computably enumerable, note that it is the domain of the function f defined by

$$f(z) = \mu y (\text{len}(z) = 2 \wedge T((z)_0, (z)_1, y)).$$

For, if $\varphi_e(x)$ is defined, $f(\langle e, x \rangle)$ finds a halting computation sequence; if $\varphi_e(x)$ is undefined, so is $f(\langle e, x \rangle)$; and if z doesn't even code a pair, then $f(z)$ is also undefined.

The fact that K_0 is not computable is just the undecidability of the halting problem, ??.

The set K_0 is the set of pairs $\langle e, x \rangle$ such that $\varphi_e(x) \downarrow$, O.s., $\langle e, x \rangle \in K_0$ iff φ_e is defined (halts) on input x , so it is also called the “halting set.” The set $K = \{e : \varphi_e(e) \downarrow\}$ is the “self-halting set.” It is often used as a canonical undecidable set.

cmp:thy:nep:
thm:K **Teorema thy.2.** *The self-halting set $K = \{e : \varphi_e(e) \downarrow\}$ is **c.e.** but not decidable.*

Prova. Suppose K is decidable, O.s., its characteristic function χ_K is computable. Let

$$d(e) = \begin{cases} 1 & \text{if } \chi_K(e) = 0 \\ \uparrow & \text{otherwise.} \end{cases}$$

Let k be the index of d , O.s., $d \simeq \varphi_k$. Then $d(k) \simeq \varphi_k(k)$. This contradicts the fact that $d(k) \downarrow$ iff $\varphi_k(k) \uparrow$, which follows from the definition of d .

K is the domain of $f(x) = \mu y T(x, x, y)$ and so is **c.e.**

Créditos das Imagens

Referências Bibliográficas