-NoValue-

thy.1 No Universal Computable Function

cmp:thy:nou:sec Although there is a partial computable function that is total for the partial computable functions, there is no total computable function that is universal for the total computable functions.

cmp:thy:nou:thm:no-univ **Teorema thy.1.** *There is no universal computable function. In other words, any function* $\mathrm{Un}'(k, x)$ *which is such that if* $f(x)$ *is a total computable function, then there is a natural number* k *such that* $f(x) = \mathrm{Un}'(k, x)$ *for every* x*, is not computable.*

Prova. The proof is a simple diagonalization: if $\mathrm{Un}'(k, x)$ were total and computable, then

$$d(x) = \mathrm{Un}'(x, x) + 1$$

would also be total and computable. However, by definition, $d(k)$ is not equal to $\mathrm{Un}'(k, k)$. Hence, for every k, the values of $d(x)$ and $\mathrm{Un}'(k, x)$ differ for at least one x, namely $x = k$. □

?? above shows that we can get around this diagonalization argument, but only at the expense of allowing the universal function to be partial. That is, Un is universal for the total computable functions, it just isn't total. The diagonalization argument doesn't work in the partial case. explanation

Problema thy.1. To understand why the diagonalization argument in the proof of Teorema thy.1 does not work in the partial case, consider the function $f(x) \simeq \mathrm{Un}(x, x) + 1$. Is it partial computable? If so, it has an index e, O.s., $f(x) \simeq \mathrm{Un}(e, x)$. What can you say about $f(e)$?

Créditos das Imagens

Referências Bibliográficas