-NoValue-

thy.1 Computably Enumerable Sets not Closed under Complement

cmp:thy:cmp:sec Suppose A is computably enumerable. Is the complement of A, $\overline{A} = \mathbb{N} \setminus A$, always computably enumerable as well? The following theorem and corollary show that the answer is "no."

cmp:thy:cmp:thm:ce-comp **Teorema thy.1.** *Let A be any set of natural numbers. Then A is computable if and only if both A and $\overline{A}$ are computably enumerable.*

Prova. The forwards direction is easy: if A is computable, then $\overline{A}$ is computable as well ($\chi_A = 1 \dot{-} \chi_{\overline{A}}$), and so both are computably enumerable.

In the other direction, suppose A and $\overline{A}$ are both computably enumerable. Let A be the domain of φ_d, and let $\overline{A}$ be the domain of φ_e. Define h by

$$h(x) = \mu s\ (T(d, x, s) \lor T(e, x, s)).$$

In other words, on input x, h searches for either a halting computation of φ_d or a halting computation of φ_e. Now, if $x \in A$, it will succeed in the first case, and if $x \in \overline{A}$, it will succeed in the second case. So, h is a total computable function. But now we have that for every x, $x \in A$ if and only if $T(e, x, h(x))$, O.s., if φ_e is the one that is defined. Since $T(e, x, h(x))$ is a computable relation, A is computable. □

It is easier to understand what is going on in informal computational terms: to decide A, on input x search for halting computations of φ_e and φ_f. One of them is bound to halt; if it is φ_e, then x is in A, and otherwise, x is in $\overline{A}$. explanation

cmp:thy:cmp:cor:comp-k **Corolário thy.2.** *$\overline{K_0}$ is not computably enumerable.*

Prova. We know that K_0 is computably enumerable, but not computable. If $\overline{K_0}$ were computably enumerable, then K_0 would be computable by Teorema thy.1, contradicting **??**. □

Créditos das Imagens

Referências Bibliográficas