

## mac.1 Unary Representation of Numbers

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Turing machines work on sequences of symbols written on their tape. Depending on the alphabet a Turing machine uses, these sequences of symbols can represent various inputs and outputs. Of particular interest, of course, are Turing machines which compute *arithmetical* functions, i.e., functions of natural numbers. A simple way to represent positive integers is by coding them as sequences of a single symbol 1. If  $n \in \mathbb{N}$ , let  $1^n$  be the empty sequence if  $n = 0$ , and otherwise the sequence consisting of exactly  $n$  1's.

explanation

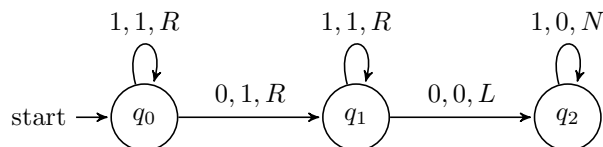
**Definition mac.1 (Computation).** A Turing machine  $M$  computes the function  $f: \mathbb{N}^n \rightarrow \mathbb{N}$  iff  $M$  halts on input

$$1^{k_1} 0 1^{k_2} 0 \dots 0 1^{k_n}$$

with output  $1^{f(k_1, \dots, k_n)}$ .

**Example mac.2. Addition:** Build a machine that, when given an input of two non-empty strings of 1's of length  $n$  and  $m$ , computes the function  $f(n, m) = n + m$ .

We want to come up with a machine that starts with two blocks of strokes on the tape and halts with one block of strokes. We first need a method to carry out. The input strokes are separated by a blank, so one method would be to write a stroke on the square containing the blank, and erase the first (or last) stroke. This would result in a block of  $n + m$  1's. Alternatively, we could proceed in a similar way to the doubler machine, by erasing a stroke from the first block, and adding one to the second block of strokes until the first block has been removed completely. We will proceed with the former example.



**Problem mac.1.** Trace through the configurations of the machine for input  $\langle 3, 5 \rangle$ .

**Problem mac.2. Subtraction:** Design a Turing machine that when given an input of two non-empty strings of strokes of length  $n$  and  $m$ , where  $n > m$ , computes the function  $f(n, m) = n - m$ .

**Problem mac.3. Equality:** Design a Turing machine to compute the following function:

$$\text{equality}(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$$

where  $x$  and  $y$  are integers greater than 0.

**Problem mac.4.** Design a Turing machine to compute the function  $\min(x, y)$  where  $x$  and  $y$  are positive integers represented on the tape by strings of 1's separated by a 0. You may use additional symbols in the alphabet of the machine.

The function  $\min$  selects the smallest value from its arguments, so  $\min(3, 5) = 3$ ,  $\min(20, 16) = 16$ , and  $\min(4, 4) = 4$ , and so on.

## Photo Credits

## Bibliography