mac.1 Disciplined Machines

tur:mac:dis: sec In section ??, we considered Turing machines that have a single, designated halting state h—such machines are guaranteed to halt, if they halt at all, in state h. In this way, machines with a single halting state are more "disciplined" than we allow Turing machines in general to be. There are other restrictions we might impose on the behavior of Turing machines. For instance, we also have not prohibited Turing machines from ever erasing the tape-end marker on square 0, or to attempt to move left from square 0. (Our definition states that the head simply stays on square 0 in this case; other definitions have the machine halt.) It is likewise sometimes desirable to be able to assume that a Turing machine, if it halts at all, halts on square 1. explanation

tur:mac:dis: defn:disciplined **Definition mac.1.** A Turing machine M is *disciplined* iff

1. it has a designated single halting state h,
2. it halts, if it halts at all, while scanning square 1,
3. it never erases the $\triangleright$ symbol on square 0, and
4. it never attempts to move left from square 0.

We have already discussed that any Turing machine can be changed into one with the same behavior but with a designated halting state. This is done simply by adding a new state h, and adding an instruction $\delta(q, \sigma) = \langle h, \sigma, N \rangle$ for any pair $\langle q, \sigma \rangle$ where the original δ is undefined. It is true, although tedious to prove, that any Turing machine M can be turned into a disciplined Turing machine M' which halts on the same inputs and produces the same output. For instance, if the Turing machine halts and is not on square 1, we can add some instructions to make the head move left until it finds the tape-end marker, then move one square to the right, then halt. We'll leave you to think about how the other conditions can be dealt with. explanation

Example mac.2. In Figure 1, we turn the addition machine from ?? into a disciplined machine.

tur:mac:dis: prop:disciplined **Proposition mac.3.** *For every Turing machine M, there is a disciplined Turing machine M' which halts with output O if M halts with output O, and does not halt if M does not halt. In particular, any function $f \colon \mathbb{N}^n \to \mathbb{N}$ computable by a Turing machine is also computable by a disciplined Turing machine.*

Problem mac.1. Give a disciplined machine that computes $f(x) = x + 1$.

Problem mac.2. Find a disciplined machine which, when started on input 1^n produces output $1^n \frown 0 \frown 1^n$.

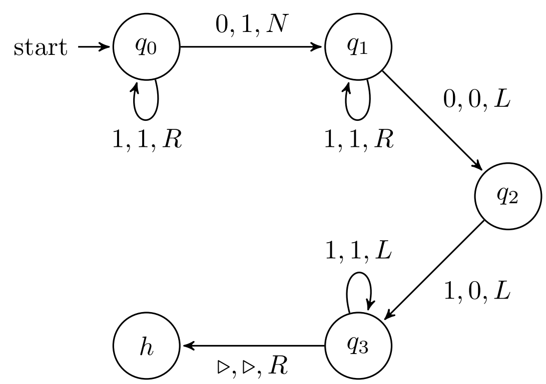

Figure 1: A disciplined addition machine

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fig:adder-disc

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Bibliography