Recall tracing through the configurations of the even machine earlier. The imaginary mechanism consisting of tape, read/write head, and Turing machine program is really just an intuitive way of visualizing what a Turing machine computation is. Formally, we can define the computation of a Turing machine on a given input as a sequence of configurations—and a configuration in turn is a sequence of symbols (corresponding to the contents of the tape at a given point in the computation), a number indicating the position of the read/write head, and a state. Using these, we can define what the Turing machine $M$ computes on a given input.

**Definition tur.1 (Configuration).** A configuration of Turing machine $M = \langle Q, \Sigma, q_0, \delta \rangle$ is a triple $\langle C, m, q \rangle$ where

1. $C \in \Sigma^*$ is a finite sequence of symbols from $\Sigma$,
2. $m \in \mathbb{N}$ is a number $< \text{len}(C)$, and
3. $q \in Q$

Intuitively, the sequence $C$ is the content of the tape (symbols of all squares from the leftmost square to the last non-blank or previously visited square), $m$ is the number of the square the read/write head is scanning (beginning with 0 being the number of the leftmost square), and $q$ is the current state of the machine.

The potential input for a Turing machine is a sequence of symbols, usually a sequence that encodes a number in some form. The initial configuration of the Turing machine is that configuration in which we start the Turing machine to work on that input: the tape contains the tape end marker immediately followed by the input written on the squares to the right, the read/write head is scanning the leftmost square of the input (i.e., the square to the right of the left end marker), and the mechanism is in the designated start state $q_0$.

**Definition tur.2 (Initial configuration).** The initial configuration of $M$ for input $I \in \Sigma^*$ is

$\langle \triangleright \ I, 1, q_0 \rangle$.

The $\triangleright$ symbol is for concatenation—the input string begins immediately to the left end marker.

**Definition tur.3.** We say that a configuration $\langle C, m, q \rangle$ yields the configuration $\langle C', m', q' \rangle$ in one step (according to $M$), iff

1. the $m$-th symbol of $C$ is $\sigma$,
2. the instruction set of $M$ specifies $\delta(q, \sigma) = \langle q', \sigma', D \rangle$,
3. the $m$-th symbol of $C'$ is $\sigma'$, and
4. a) $D = L$ and $m' = m - 1$ if $m > 0$, otherwise $m' = 0$, or
   b) $D = R$ and $m' = m + 1$, or
   c) $D = N$ and $m' = m$,

5. if $m' = \text{len}(C)$, then $\text{len}(C') = \text{len}(C) + 1$ and the $m'$-th symbol of $C'$ is 0. Otherwise $\text{len}(C') = \text{len}(C)$.

6. for all $i$ such that $i < \text{len}(C)$ and $i \neq m$, $C'(i) = C(i)$,

**Definition tur.4.** A run of $M$ on input $I$ is a sequence $C_i$ of configurations of $M$, where $C_0$ is the initial configuration of $M$ for input $I$, and each $C_i$ yields $C_{i+1}$ in one step.

We say that $M$ halts on input $I$ after $k$ steps if $C_k = \langle C, m, q \rangle$, the $m$th symbol of $C$ is $\sigma$, and $\delta(q, \sigma)$ is undefined. In that case, the output of $M$ for input $I$ is $O$, where $O$ is a string of symbols not ending in 0 such that $C = \triangleright \leadsto O \leftarrow 0^j$ for some $j \in \mathbb{N}$. ($0^j$ is a sequence of $j$ 0’s.)

According to this definition, the output $O$ of $M$ always ends in a symbol other than 0, or, if at time $k$ the entire tape is filled with 0 (except for the leftmost $\triangleright$), $O$ is the empty string.

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**Bibliography**