

## mac.1 The Church–Turing Thesis

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Turing machines are supposed to be a precise replacement for the concept of an effective procedure. Turing thought that anyone who grasped both the concept of an effective procedure and the concept of a Turing machine would have the intuition that anything that could be done via an effective procedure could be done by Turing machine. This claim is given support by the fact that all the other proposed precise replacements for the concept of an effective procedure turn out to be extensionally equivalent to the concept of a Turing machine—that is, they can compute exactly the same set of functions. This claim is called the *Church–Turing thesis*.

**Definition mac.1 (Church–Turing thesis).** The *Church–Turing Thesis* states that anything computable via an effective procedure is Turing computable.

The Church–Turing thesis is appealed to in two ways. The first kind of use of the Church–Turing thesis is an excuse for laziness. Suppose we have a description of an effective procedure to compute something, say, in “pseudo-code.” Then we can invoke the Church–Turing thesis to justify the claim that the same function is computed by some Turing machine, even if we have not in fact constructed it.

The other use of the Church–Turing thesis is more philosophically interesting. It can be shown that there are functions which cannot be computed by Turing machines. From this, using the Church–Turing thesis, one can conclude that it cannot be effectively computed, using any procedure whatsoever. For if there were such a procedure, by the Church–Turing thesis, it would follow that there would be a Turing machine for it. So if we can prove that there is no Turing machine that computes it, there also can’t be an effective procedure. In particular, the Church–Turing thesis is invoked to claim that the so-called halting problem not only cannot be solved by Turing machines, it cannot be effectively solved at all.

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## Bibliography