

set.1 Some Important Sets

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Example set.1. We will mostly be dealing with sets whose **elements** are mathematical objects. Four such sets are important enough to have specific names:

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

the set of natural numbers

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

the set of integers

$$\mathbb{Q} = \{m/n : m, n \in \mathbb{Z} \text{ and } n \neq 0\}$$

the set of rationals

$$\mathbb{R} = (-\infty, \infty)$$

the set of real numbers (the continuum)

These are all *infinite* sets, that is, they each have infinitely many **elements**.

As we move through these sets, we are adding *more* numbers to our stock. Indeed, it should be clear that $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$: after all, every natural number is an integer; every integer is a rational; and every rational is a real. Equally, it should be clear that $\mathbb{N} \subsetneq \mathbb{Z} \subsetneq \mathbb{Q}$, since -1 is an integer but not a natural number, and $1/2$ is rational but not integer. It is less obvious that $\mathbb{Q} \subsetneq \mathbb{R}$, i.e., that there are some real numbers which are not rational.

We'll sometimes also use the set of positive integers $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ and the set containing just the first two natural numbers $\mathbb{B} = \{0, 1\}$.

Example set.2 (Strings). Another interesting example is the set A^* of *finite strings* over an alphabet A : any finite sequence of elements of A is a string over A . We include the *empty string* Λ among the strings over A , for every alphabet A . For instance,

$$\mathbb{B}^* = \{\Lambda, 0, 1, 00, 01, 10, 11, \\ 000, 001, 010, 011, 100, 101, 110, 111, 0000, \dots\}.$$

If $x = x_1 \dots x_n \in A^*$ is a string consisting of n “letters” from A , then we say *length* of the string is n and write $\text{len}(x) = n$.

Example set.3 (Infinite sequences). For any set A we may also consider the set A^ω of infinite sequences of **elements** of A . An infinite sequence $a_1 a_2 a_3 a_4 \dots$ consists of a one-way infinite list of objects, each one of which is **an element** of A .

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Bibliography