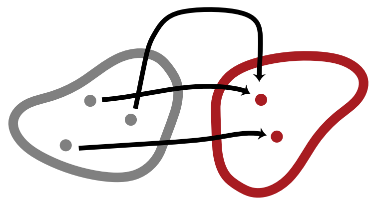

Figure 1: A surjective function has every element of the codomain as a value.

sfr:fun:kin: fig:surjective

Figure 2: An injective function never maps two different arguments to the same value.

sfr:fun:kin: fig:injective

fun.1 Kinds of Functions

sfr:fun:kin: sec

It will be useful to introduce a kind of taxonomy for some of the kinds of functions which we encounter most frequently. explanation

To start, we might want to consider functions which have the property that every member of the codomain is a value of the function. Such functions are called surjective, and can be pictured as in Figure 1.

Definition fun.1 (Surjective function). A function $f\colon A \to B$ is *surjective* iff B is also the range of f, i.e., for every $y \in B$ there is at least one $x \in A$ such that $f(x) = y$, or in symbols:

$$(\forall y \in B)(\exists x \in A) f(x) = y.$$

We call such a function a surjection from A to B.

If you want to show that f is a surjection, then you need to show that every object in f's codomain is the value of $f(x)$ for some input x. explanation

Note that any function *induces* a surjection. After all, given a function $f\colon A \to B$, let $f'\colon A \to \operatorname{ran}(f)$ be defined by $f'(x) = f(x)$. Since $\operatorname{ran}(f)$ is *defined* as $\{f(x) \in B : x \in A\}$, this function f' is guaranteed to be a surjection

Now, any function maps each possible input to a unique output. But there are also functions which never map different inputs to the same outputs. Such functions are called injective, and can be pictured as in Figure 2. explanation

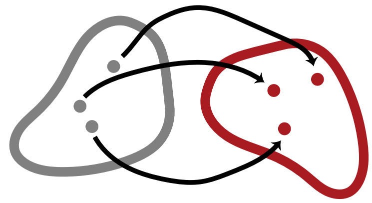

Figure 3: A bijective function uniquely pairs the elements of the codomain with those of the domain.

sfr:fun:kin: fig:bijective

Definition fun.2 (Injective function). A function $f \colon A \to B$ is *injective* iff for each $y \in B$ there is at most one $x \in A$ such that $f(x) = y$. We call such a function an injection from A to B.

explanation

If you want to show that f is an injection, you need to show that for any elements x and y of f's domain, if $f(x) = f(y)$, then $x = y$.

Example fun.3. The constant function $f \colon \mathbb{N} \to \mathbb{N}$ given by $f(x) = 1$ is neither injective, nor surjective.

The identity function $f \colon \mathbb{N} \to \mathbb{N}$ given by $f(x) = x$ is both injective and surjective.

The successor function $f \colon \mathbb{N} \to \mathbb{N}$ given by $f(x) = x + 1$ is injective but not surjective.

The function $f \colon \mathbb{N} \to \mathbb{N}$ defined by:

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{x+1}{2} & \text{if } x \text{ is odd.} \end{cases}$$

is surjective, but not injective.

explanation

Often enough, we want to consider functions which are both injective and surjective. We call such functions bijective. They look like the function pictured in Figure 3. Bijections are also sometimes called *one-to-one correspondences*, since they uniquely pair elements of the codomain with elements of the domain.

Definition fun.4 (Bijection). A function $f \colon A \to B$ is *bijective* iff it is both surjective and injective. We call such a function a bijection from A to B (or between A and B).

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Bibliography