z.1 Union

?? gave us intersections. But if we want arbitrary unions to exist, we need to lay down another axiom:

Axiom (Union). For any set $A$, the set $\bigcup A = \{x : (\exists b \in A)x \in b\}$ exists.

$$\forall A \exists U \forall x (x \in U \leftrightarrow (\exists b \in A)x \in b)$$

This axiom is also justified by the cumulative-iterative conception. Let $A$ be a set, so $A$ is formed at some stage $S$ (by Stages-are-key). Every member of $A$ was formed before $S$ (by Stages-accumulate); so, reasoning similarly, every member of every member of $A$ was formed before $S$. Thus all of those sets are available before $S$, to be formed into a set at $S$. And that set is just $\bigcup A$.

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Bibliography