

## z.1 $\mathbf{Z}^-$ : a Milestone

sth:z:milestone:  
sec We will revisit *Stages-hit-infinity* in the next section. However, with the Axiom of Infinity, we have reached an important milestone. We now have all the axioms required for the theory  $\mathbf{Z}^-$ . In detail:

**Definition z.1.** The theory  $\mathbf{Z}^-$  has these axioms: Extensionality, Union, Pairs, Powersets, Infinity, and all instances of the Separation scheme.

The name stands for *Zermelo* set theory (*minus* something which we will come to later). Zermelo deserves the honour, since he essentially formulated this theory in his 1908.<sup>1</sup>

This theory is powerful enough to allow us to do an enormous amount of mathematics. In particular, you *should* look back through ??, and convince yourself that everything we did, naïvely, could be done more formally within  $\mathbf{Z}^-$ . (Once you have done that for a bit, you might want to skip ahead and read ??.) So, henceforth, and without any further comment, we will take ourselves to be working in  $\mathbf{Z}^-$  (at least).

## Photo Credits

## Bibliography

Potter, Michael. 2004. *Set Theory and its Philosophy*. Oxford: Oxford University Press.

Zermelo, Ernst. 1908. Untersuchungen über die Grundlagen der Mengenlehre I. *Mathematische Annalen* 65: 261–81.

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<sup>1</sup>For interesting comments on the history and technicalities, see [Potter \(2004, Appendix A\)](#).