story.1  Extensionality

The very first thing to say is that sets are individuated by their elements. More precisely:

**Axiom (Extensionality).** If sets \( A \) and \( B \) have the same elements, then \( A \) and \( B \) are the same set.

\[
\forall A \forall B \left( \forall x \left( x \in A \leftrightarrow x \in B \right) \rightarrow A = B \right)
\]

We assumed this throughout ???. But it bears repeating. The Axiom of Extensionality expresses the basic idea that a set is determined by its elements. (So sets might be contrasted with concepts, where precisely the same objects might fall under many different concepts.)

Why embrace this principle? Well, it is plausible to say that any denial of Extensionality is a decision to abandon anything which might even be called set theory. Set theory is no more nor less than the theory of extensional collections.

The real challenge in ??, though, is to lay down principles which tell us which sets exist. And it turns out that the only truly “obvious” answer to this question is provably wrong.

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Bibliography