

replacement.1 Limitation-of-size

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sec

Perhaps the most common attempt to offer an “intrinsic” justification of Replacement comes via the following notion:

Limitation-of-size. Any things form a set, provided that there are not too many of them.

This principle will immediately vindicate Replacement. After all, any set formed by Replacement cannot be any larger than any set from which it was formed. Stated precisely: suppose you form a set $\tau[A] = \{\tau(x) : x \in A\}$ using Replacement; then $\tau[A] \preceq A$; so if the elements of A were not too numerous to form a set, their images are not too numerous to form $\tau[A]$.

The obvious difficulty with invoking *Limitation-of-size* to justify Replacement is that we have *not* yet laid down any principle like *Limitation-of-size*. Moreover, when we told our story about the cumulative-iterative conception of set in ??-??, nothing ever *hinted* in the direction of *Limitation-of-size*. This, indeed, is precisely why Boolos at one point wrote: “Perhaps one may conclude that there are at least two thoughts ‘behind’ set theory” (1989, p. 19). On the one hand, the ideas surrounding the cumulative-iterative conception of set are meant to vindicate **Z**. On the other hand, *Limitation-of-size* is meant to vindicate Replacement.

But the issue it is not just that we have thus far been *silent* about *Limitation-of-size*. Rather, the issue is that *Limitation-of-size* (as just formulated) seems to sit quite badly with the cumulative-iterative notion of set. After all, it mentions nothing about the idea of sets as formed in *stages*.

This is really not much of a surprise, given the history of these “two thoughts” (i.e., the cumulative-iterative conception of set, and *Limitation-of-size*). These “two thoughts” ultimately amount to two rather different projects for blocking the set-theoretic paradoxes. The cumulative-iterative notion of set blocks Russell’s paradox by saying, roughly: *we should never have expected a Russell set to exist, because it would not be “formed” at any stage*. By contrast, *Limitation-of-size* is meant to rule out the Russell set, by saying, roughly: *we should never have expected a Russell set to exist, because it would have been too big*.

Put like this, then, let’s be blunt: considered as a reply to the paradoxes, *Limitation-of-size* stands in need of *much* more justification. Consider, for example, this version of Russell’s Paradox: *no pug sniffs exactly the pugs which don’t sniff themselves* (see ??). If you ask “why is there no such pug?”, it is not a good answer to be told that such a pug would have to sniff too many pugs. So why would it be a good intuitive explanation, of the non-existence of a Russell set, that it would have to be “too big” to exist?

In short, it’s forgivable if you are a bit mystified concerning the “intuitive” motivation for *Limitation-of-size*.

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Bibliography

Boolos, George. 1989. Iteration again. *Philosophical Topics* 17(2): 5–21.