

ordinals.1 \mathbf{ZF}^- : a milestone

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The question of how to justify Replacement (if at all) is not straightforward. As such, we will reserve that for ???. However, with the addition of Replacement, we have reached another important milestone. We now have all the axioms required for the theory \mathbf{ZF}^- . In detail:

Definition ordinals.1. The theory \mathbf{ZF}^- has these axioms: Extensionality, Union, Pairs, Powersets, Infinity, and all instances of the Separation and Replacement schemes. Otherwise put, \mathbf{ZF}^- adds Replacement to \mathbf{Z}^- .

This stands for *Zermelo–Fraenkel* set theory (*minus* something which we will come to later). Fraenkel gets the honour, since he is credited with the formulation of Replacement in 1922, although the first precise formulation was due to Skolem (1922).

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Bibliography

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