

ordinals.1 The General Idea of an Ordinal

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Consider the natural numbers, in their usual order:

$$0 < 1 < 2 < 3 < 4 < 5 < \dots$$

We call this, in the jargon, an ω -sequence. And indeed, this general ordering is mirrored in our initial construction of the stages of the set hierarchy. But, now suppose we move 0 to the end of this sequence, so that it comes after all the other numbers:

$$1 < 2 < 3 < 4 < 5 < \dots < 0$$

We have the same entities here, but ordered in a fundamentally different way: our first ordering had no last element; our new ordering does. Indeed, our new ordering consists of an ω -sequence of entities $(1, 2, 3, 4, 5, \dots)$, followed by another entity. It will be an $\omega + 1$ -sequence.

We can generate even more types of ordering, using just these entities. For example, consider all the even numbers (in their natural order) followed by all the odd numbers (in their natural order):

$$0 < 2 < 4 < \dots < 1 < 3 < \dots$$

This is an ω -sequence followed by another ω -sequence; an $\omega + \omega$ -sequence.

Well, we can keep going. But what we would like is a general way to understand this talk about *orderings*.

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Bibliography