

## syn.1 Semantic Notions

pl:syn:sem:  
sec We define the following semantic notions:

- Definition syn.1.**
1. A formula  $\varphi$  is *satisfiable* if for some  $\mathbf{v}$ ,  $\mathbf{v} \models \varphi$ ; it is *unsatisfiable* if for no  $\mathbf{v}$ ,  $\mathbf{v} \models \varphi$ ;
  2. A formula  $\varphi$  is a *tautology* if  $\mathbf{v} \models \varphi$  for all valuations  $v$ ;
  3. A formula  $\varphi$  is *contingent* if it is satisfiable but not a tautology;
  4. If  $\Gamma$  is a set of formulas,  $\Gamma \models \varphi$  (“ $\Gamma$  entails  $\varphi$ ”) if and only if  $\mathbf{v} \models \varphi$  for every valuation  $\mathbf{v}$  for which  $\mathbf{v} \models \Gamma$ .
  5. If  $\Gamma$  is a set of formulas,  $\Gamma$  is *satisfiable* if there is a valuation  $\mathbf{v}$  for which  $\mathbf{v} \models \Gamma$ , and  $\Gamma$  is *unsatisfiable* otherwise.

pl:syn:sem:  
prop:semanticalfacts **Proposition syn.2.**

1.  $\varphi$  is a tautology if and only if  $\emptyset \models \varphi$ ;
2. If  $\Gamma \models \varphi$  and  $\Gamma \models \varphi \rightarrow \psi$  then  $\Gamma \models \psi$ ;
3. If  $\Gamma$  is satisfiable then every finite subset of  $\Gamma$  is also satisfiable;
4. *Monotony:* if  $\Gamma \subseteq \Delta$  and  $\Gamma \models \varphi$  then also  $\Delta \models \varphi$ ;
5. *Transitivity:* if  $\Gamma \models \varphi$  and  $\Delta \cup \{\varphi\} \models \psi$  then  $\Gamma \cup \Delta \models \psi$ ;

*Proof.* Exercise. □

**Problem syn.1.** Prove [Proposition syn.2](#)

pl:syn:sem:  
prop:entails-unsat **Proposition syn.3.**  $\Gamma \models \varphi$  if and only if  $\Gamma \cup \{\neg\varphi\}$  is unsatisfiable;

*Proof.* Exercise. □

**Problem syn.2.** Prove [Proposition syn.3](#)

pl:syn:sem:  
thm:sem-deduction **Theorem syn.4 (Semantic Deduction Theorem).**  $\Gamma \models \varphi \rightarrow \psi$  if and only if  $\Gamma \cup \{\varphi\} \models \psi$ .

*Proof.* Exercise. □

**Problem syn.3.** Prove [Theorem syn.4](#)

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## Bibliography