nor.1 The Normalization Theorem

pt:nor:thm:sec

A derivation is in *normal* form if it has no cut segments. Clearly this is the case iff we can apply neither a permutation conversion nor a reduction conversion to it. To *normalize* a derivation means to transform it into a normal proof by applying permutation and reduction conversions until no cuts remain. That this is always possible is the *normalization theorem*.

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Theorem nor.1. *Any* **N1i**-*derivation* δ *normalizes, i.e., can be transformed into a derivation* δ' *without cuts.*

Proof. By induction on the cut rank and length of δ. The induction hypothesis is that any derivation which either has lower cut rank, or the same cut rank but lower cut length, normalizes.

If the cut length is 0, there are no cuts, δ is already normal. So assume that cut rank and cut length of δ are > 0. Pick a topmost, rightmost maximal cut segment. If the cut segment has length > 1, apply a permutation conversion. If the cut segment has length 1, apply the corresponding reduction conversion. This results in a new derivation which either has the same cut rank but a lower cut length, or a lower cut rank, and the inductive hypothesis applies. □

The strategy used in the above proof (always deal with a topmost rightmost maximal cut) suggests a specific order in which conversions must be applied to a derivation in order to result in a normal derivation. Surprisingly enough, the order actually does not matter. Applying permutation and reduction conversions in any order eventually results in a normal proof.

The definition of segment and cut can easily be formulated for **N2i**, and the permutation and reduction conversions for **N1i** directly translate to **N2i**. Consequently, the normalization theorem also holds for **N2i**.

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Theorem nor.2. *Any* **N2i**-*derivation* δ *normalizes, i.e., can be transformed into a derivation* δ' *without cuts.*

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Bibliography