

$\frac{n \text{ T } \Box \varphi}{m \text{ T } \varphi} \Box \text{T}$ <p><math>m</math> is used</p>	$\frac{n \text{ F } \Box \varphi}{m \text{ F } \varphi} \Box \text{F}$ <p><math>m</math> is new</p>
$\frac{n \text{ T } \Diamond \varphi}{m \text{ T } \varphi} \Diamond \text{T}$ <p><math>m</math> is new</p>	$\frac{n \text{ F } \Diamond \varphi}{m \text{ F } \varphi} \Diamond \text{F}$ <p><math>m</math> is used</p>

Table 1: Simplified rules for **S5**.

nml:tab:s5:  
tab:rules-S5

### tab.1 Simple Tableaux for S5

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sec

**S5** is sound and complete with respect to the class of universal models, i.e., models where every world is accessible from every world. In universal models the accessibility relation doesn't matter: "there is a world  $w$  where  $\mathfrak{M}, w \Vdash \varphi$ " is true if and only if there is such a  $w$  that's accessible from  $u$ . So in **S5**, we can define models as simply a set of worlds and a valuation  $V$ . This suggests that we should be able to simplify the **tableau** rules as well. In the general case, we take as prefixes sequences of positive integers, so that we can keep track of which such prefixes name worlds which are accessible from others:  $\sigma.n$  names a world accessible from  $\sigma$ . But in **S5** any world is accessible from any world, so there is no need to so keep track. Instead, we can use positive integers as prefixes. The simplified rules are given in **Table 1**.

**Example tab.1.** We give a simplified closed tableau that shows **S5**  $\vdash 5$ , i.e.,  $\Diamond \varphi \rightarrow \Box \Diamond \varphi$ .

1.	$1 \text{ F } \Diamond \varphi \rightarrow \Box \Diamond \varphi$	Assumption
2.	$1 \text{ T } \Diamond \varphi$	$\rightarrow \text{F } 1$
3.	$1 \text{ F } \Box \Diamond \varphi$	$\rightarrow \text{F } 1$
4.	$2 \text{ F } \Diamond \varphi$	$\Box \text{F } 3$
5.	$3 \text{ T } \varphi$	$\Diamond \text{T } 2$
6.	$3 \text{ F } \varphi$	$\Diamond \text{F } 4$
	$\otimes$	

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**Bibliography**