$\dfrac{\sigma\,\mathbb{T}\,\neg\varphi}{\sigma\,\mathbb{F}\,\varphi}\ \neg\mathbb{T}$	$\dfrac{\sigma\,\mathbb{F}\,\neg\varphi}{\sigma\,\mathbb{T}\,\varphi}\ \neg\mathbb{F}$
$\dfrac{\sigma\,\mathbb{T}\,\varphi\wedge\psi}{\begin{array}{l}\sigma\,\mathbb{T}\,\varphi\\ \sigma\,\mathbb{T}\,\psi\end{array}}\ \wedge\mathbb{T}$	$\dfrac{\sigma\,\mathbb{F}\,\varphi\wedge\psi}{\sigma\,\mathbb{F}\,\varphi \quad\mid\quad \sigma\,\mathbb{F}\,\psi}\ \wedge\mathbb{F}$
$\dfrac{\sigma\,\mathbb{T}\,\varphi\vee\psi}{\sigma\,\mathbb{T}\,\varphi \quad\mid\quad \sigma\,\mathbb{T}\,\psi}\ \vee\mathbb{T}$	$\dfrac{\sigma\,\mathbb{F}\,\varphi\vee\psi}{\begin{array}{l}\sigma\,\mathbb{F}\,\varphi\\ \sigma\,\mathbb{F}\,\psi\end{array}}\ \vee\mathbb{F}$
$\dfrac{\sigma\,\mathbb{T}\,\varphi\to\psi}{\sigma\,\mathbb{F}\,\varphi \quad\mid\quad \sigma\,\mathbb{T}\,\psi}\ \to\mathbb{T}$	$\dfrac{\sigma\,\mathbb{F}\,\varphi\to\psi}{\begin{array}{l}\sigma\,\mathbb{T}\,\varphi\\ \sigma\,\mathbb{F}\,\psi\end{array}}\ \to\mathbb{F}$

Table 1: Prefixed tableau rules for the propositional connectives

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tab:prop-rules

tab.1 Rules for K

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sec

The rules for the regular propositional connectives are the same as for regular propositional signed tableaux, just with prefixes added. In each case, the rule applied to a signed formula $\sigma\,S\,\varphi$ produces new formulas that are also prefixed by σ. This should be intuitively clear: e.g., if $\varphi\wedge\psi$ is true at (a world named by) σ, then φ and ψ are true at σ (and not at any other world). We collect the propositional rules in Table 1.

The closure condition is the same as for ordinary tableaux, although we require that not just the formulas but also the prefixes must match. So a branch is closed if it contains both

$$\sigma\,\mathbb{T}\,\varphi \quad\text{and}\quad \sigma\,\mathbb{F}\,\varphi$$

for some prefix σ and formula φ.

The rules for setting up assumptions is also as for ordinary tableaux, except that for assumptions we always use the prefix 1. (It does not matter which prefix we use, as long as it's the same for all assumptions.) So, e.g., we say that

$$\psi_1,\ldots,\psi_n\vdash\varphi$$

iff there is a closed tableau for the assumptions

$$1\,\mathbb{T}\,\psi_1,\ldots,1\,\mathbb{T}\,\psi_n,1\,\mathbb{F}\,\varphi.$$

For the modal operators $\Box$ and $\Diamond$, the prefix of the conclusion of the rule applied to a formula with prefix σ is $\sigma.n$. However, which n is allowed depends on whether the sign is $\mathbb{T}$ or $\mathbb{F}$.

$\dfrac{\sigma\,\mathbb{T}\,\Box\varphi}{\sigma.n\,\mathbb{T}\,\varphi}\,\Box\mathbb{T}$ $\sigma.n$ is used	$\dfrac{\sigma\,\mathbb{F}\,\Box\varphi}{\sigma.n\,\mathbb{F}\,\varphi}\,\Box\mathbb{F}$ $\sigma.n$ is new
$\dfrac{\sigma\,\mathbb{T}\,\Diamond\varphi}{\sigma.n\,\mathbb{T}\,\varphi}\,\Diamond\mathbb{T}$ $\sigma.n$ is new	$\dfrac{\sigma\,\mathbb{F}\,\Diamond\varphi}{\sigma.n\,\mathbb{F}\,\varphi}\,\Diamond\mathbb{F}$ $\sigma.n$ is used

Table 2: The modal rules for K.

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tab:rules-K

The $\Box\mathbb{T}$ rule extends a branch containing $\sigma\,\mathbb{T}\,\Box\varphi$ by $\sigma.n\,\mathbb{T}\,\varphi$. Similarly, the $\Diamond\mathbb{F}$ rule extends a branch containing $\sigma\,\mathbb{F}\,\Diamond\varphi$ by $\sigma.n\,\mathbb{F}\,\varphi$. They can only be applied for a prefix $\sigma.n$ which *already* occurs on the branch in which it is applied. Let's call such a prefix "used" (on the branch).

The $\Box\mathbb{F}$ rule extends a branch containing $\sigma\,\mathbb{F}\,\Box\varphi$ by $\sigma.n\,\mathbb{F}\,\varphi$. Similarly, the $\Diamond\mathbb{T}$ rule extends a branch containing $\sigma\,\mathbb{T}\,\Diamond\varphi$ by $\sigma.n\,\mathbb{T}\,\varphi$. These rules, however, can only be applied for a prefix $\sigma.n$ which *does not* already occur on the branch in which it is applied. We call such prefixes "new" (to the branch).

The rules are given in Table 2.

The requirement that the restriction that the prefix for $\Box\mathbb{T}$ must be used is necessary as otherwise we would count the following as a closed tableau:

1.	$1\,\mathbb{T}\ \Box\varphi$	Assumption
2.	$1\,\mathbb{F}\ \Diamond\varphi$	Assumption
3.	$1.1\,\mathbb{T}\ \varphi$	$\Box\mathbb{T}\,1$
4.	$1.1\,\mathbb{F}\ \varphi$	$\Diamond\mathbb{F}\,2$
	$\otimes$	

But $\Box\varphi \not\models \Diamond\varphi$, so our proof system would be unsound. Likewise, $\Diamond\varphi \not\models \Box\varphi$, but without the restriction that the prefix for $\Box\mathbb{F}$ must be new, this would be a closed tableau:

1.	$1\,\mathbb{T}\ \Diamond\varphi$	Assumption
2.	$1\,\mathbb{F}\ \Box\varphi$	Assumption
3.	$1.1\,\mathbb{T}\ \varphi$	$\Diamond\mathbb{T}\,1$
4.	$1.1\,\mathbb{F}\ \varphi$	$\Box\mathbb{F}\,2$
	$\otimes$	

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Bibliography