

$\frac{\sigma \mathbf{T} \Box \varphi}{\sigma \mathbf{T} \varphi} \mathbf{T} \Box$	$\frac{\sigma \mathbf{F} \Diamond \varphi}{\sigma \mathbf{F} \varphi} \mathbf{T} \Diamond$
$\frac{\sigma \mathbf{T} \Box \varphi}{\sigma \mathbf{T} \Diamond \varphi} \mathbf{D} \Box$	$\frac{\sigma \mathbf{F} \Diamond \varphi}{\sigma \mathbf{F} \Box \varphi} \mathbf{D} \Diamond$
$\frac{\sigma.n \mathbf{T} \Box \varphi}{\sigma \mathbf{T} \varphi} \mathbf{B} \Box$	$\frac{\sigma.n \mathbf{F} \Diamond \varphi}{\sigma \mathbf{F} \varphi} \mathbf{B} \Diamond$
$\frac{\sigma \mathbf{T} \Box \varphi}{\sigma.n \mathbf{T} \Box \varphi} 4 \Box$	$\frac{\sigma \mathbf{F} \Diamond \varphi}{\sigma.n \mathbf{F} \Diamond \varphi} 4 \Diamond$
$\sigma.n$ is used	$\sigma.n$ is used
$\frac{\sigma.n \mathbf{T} \Box \varphi}{\sigma \mathbf{T} \Box \varphi} 4r \Box$	$\frac{\sigma.n \mathbf{F} \Diamond \varphi}{\sigma \mathbf{F} \Diamond \varphi} 4r \Diamond$

Table 1: More modal rules.

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tab:more-rules

Logic	$R$ is ...	Rules
<b>T = KT</b>	reflexive	$\mathbf{T} \Box, \mathbf{T} \Diamond$
<b>D = KD</b>	serial	$\mathbf{D} \Box, \mathbf{D} \Diamond$
<b>K4</b>	transitive	$4 \Box, 4 \Diamond$
<b>B = KTB</b>	reflexive, symmetric	$\mathbf{T} \Box, \mathbf{T} \Diamond$ $\mathbf{B} \Box, \mathbf{B} \Diamond$
<b>S4 = KT4</b>	reflexive, transitive	$\mathbf{T} \Box, \mathbf{T} \Diamond,$ $4 \Box, 4 \Diamond$
<b>S5 = KT4B</b>	reflexive, transitive, euclidean	$\mathbf{T} \Box, \mathbf{T} \Diamond,$ $4 \Box, 4 \Diamond,$ $4r \Box, 4r \Diamond$

Table 2: Tableau rules for various modal logics.

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tab:logics-rules

## tab.1 Rules for Other Accessibility Relations

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sec In order to deal with logics determined by special accessibility relations, we consider the additional rules in [Table 1](#).

Adding these rules results in systems that are sound and complete for the logics given in [Table 2](#).

**Example tab.1.** We give a closed tableau that shows  $\mathbf{S5} \vdash 5$ , i.e.,  $\Box\varphi \rightarrow \Box\Diamond\varphi$ .

1.	$1\mathbb{F} \Box\varphi \rightarrow \Box\Diamond\varphi$	Assumption
2.	$1\mathbb{T} \Box\varphi$	$\rightarrow\mathbb{F} 1$
3.	$1\mathbb{F} \Box\Diamond\varphi$	$\rightarrow\mathbb{F} 1$
4.	$1.1\mathbb{F} \Diamond\varphi$	$\Box\mathbb{F} 3$
5.	$1\mathbb{F} \Diamond\varphi$	$4r\Diamond 4$
6.	$1.1\mathbb{F} \varphi$	$\Diamond\mathbb{F} 5$
7.	$1.1\mathbb{T} \varphi$	$\Box\mathbb{T} 2$
	$\otimes$	

**Problem tab.1.** Give closed tableaux that show the following:

1.  $\mathbf{KT5} \vdash B$ ;
2.  $\mathbf{KT5} \vdash 4$ ;
3.  $\mathbf{KDB4} \vdash T$ ;
4.  $\mathbf{KB4} \vdash 5$ ;
5.  $\mathbf{KB5} \vdash 4$ ;
6.  $\mathbf{KT} \vdash D$ .

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**Bibliography**