

$\frac{\sigma \text{T}\Box\varphi}{\sigma \text{T}\varphi} \text{T}\Box$	$\frac{\sigma \text{F}\Diamond\varphi}{\sigma \text{F}\varphi} \text{T}\Diamond$
$\frac{\sigma \text{T}\Box\varphi}{\sigma \text{T}\Diamond\varphi} \text{D}\Box$	$\frac{\sigma \text{F}\Diamond\varphi}{\sigma \text{F}\Box\varphi} \text{D}\Diamond$
$\frac{\sigma.n \text{T}\Box\varphi}{\sigma \text{T}\varphi} \text{B}\Box$	$\frac{\sigma.n \text{F}\Diamond\varphi}{\sigma \text{F}\varphi} \text{B}\Diamond$
$\frac{\sigma \text{T}\Box\varphi}{\sigma.n \text{T}\Box\varphi} 4\Box$ $\sigma.n$ is used	$\frac{\sigma \text{F}\Diamond\varphi}{\sigma.n \text{F}\Diamond\varphi} 4\Diamond$ $\sigma.n$ is used
$\frac{\sigma.n \text{T}\Box\varphi}{\sigma \text{T}\Box\varphi} 4r\Box$	$\frac{\sigma.n \text{F}\Diamond\varphi}{\sigma \text{F}\Diamond\varphi} 4r\Diamond$

Table 1: More modal rules.

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tab:more-rules

Logic	R is ...	Rules
T = KT	reflexive	$\text{T}\Box, \text{T}\Diamond$
D = KD	serial	$\text{D}\Box, \text{D}\Diamond$
K4	transitive	$4\Box, 4\Diamond$
B = KTB	reflexive, symmetric	$\text{T}\Box, \text{T}\Diamond$ $\text{B}\Box, \text{B}\Diamond$
S4 = KT4	reflexive, transitive	$\text{T}\Box, \text{T}\Diamond,$ $4\Box, 4\Diamond$
S5 = KT4B	reflexive, transitive, euclidean	$\text{T}\Box, \text{T}\Diamond,$ $4\Box, 4\Diamond,$ $4r\Box, 4r\Diamond$

Table 2: Tableau rules for various modal logics.

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tab:logics-rules

tab.1 Rules for Other Accessibility Relations

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sec In order to deal with logics determined by special accessibility relations, we consider the additional rules in [table 1](#).

Adding these rules results in systems that are sound and complete for the logics given in [table 2](#).

Example tab.1. We give a closed tableau that shows $\mathbf{S5} \vdash 5$, i.e., $\Box\varphi \rightarrow \Box\Diamond\varphi$.

1.	$1\mathbb{F} \Box\varphi \rightarrow \Box\Diamond\varphi$	Assumption
2.	$1\mathbb{T} \Box\varphi$	$\rightarrow\mathbb{F} 1$
3.	$1\mathbb{F} \Box\Diamond\varphi$	$\rightarrow\mathbb{F} 1$
4.	$1.1\mathbb{F} \Diamond\varphi$	$\Box\mathbb{F} 3$
5.	$1\mathbb{F} \Diamond\varphi$	$4r\Diamond 4$
6.	$1.1\mathbb{F} \varphi$	$\Diamond\mathbb{F} 5$
7.	$1.1\mathbb{T} \varphi$	$\Box\mathbb{T} 2$
	\otimes	

Problem tab.1. Give closed tableaux that show the following:

1. $\mathbf{KT5} \vdash B$;
2. $\mathbf{KT5} \vdash 4$;
3. $\mathbf{KDB4} \vdash T$;
4. $\mathbf{KB4} \vdash 5$;
5. $\mathbf{KB5} \vdash 4$;
6. $\mathbf{KT} \vdash D$.

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Bibliography