

tab.1 Introduction

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sec **Tableaux** are certain (downward-branching) trees of **signed formulas**, i.e., pairs consisting of a truth value sign (\mathbb{T} or \mathbb{F}) and a **sentence**

$$\mathbb{T}\varphi \text{ or } \mathbb{F}\varphi.$$

A **tableau** begins with a number of *assumptions*. Each further **signed formula** is generated by applying one of the inference rules. Some inference rules add one or more **signed formulas** to a tip of the tree; others add two new tips, resulting in two branches. Rules result in **signed formulas** where the **formula** is less complex than that of the **signed formula** to which it was applied. When a branch contains both $\mathbb{T}\varphi$ and $\mathbb{F}\varphi$, we say the branch is *closed*. If every branch in a **tableau** is closed, the entire **tableau** is closed. A closed **tableau** constitutes a **derivation** that shows that the set of **signed formulas** which were used to begin the **tableau** are unsatisfiable. This can be used to define a \vdash relation: $\Gamma \vdash \varphi$ iff there is some finite set $\Gamma_0 = \{\psi_1, \dots, \psi_n\} \subseteq \Gamma$ such that there is a closed **tableau** for the assumptions

$$\{\mathbb{F}\varphi, \mathbb{T}\psi_1, \dots, \mathbb{T}\psi_n\}.$$

For modal logics, we have to both extend the notion of **signed formula** and add rules that cover \Box and \Diamond . In addition to a sign (\mathbb{T} or \mathbb{F}), **formulas** in modal **tableaux** also have *prefixes* σ . The prefixes are non-empty sequences of positive integers, i.e., $\sigma \in (\mathbb{Z}^+)^* \setminus \{\Lambda\}$. When we write such prefixes without the surrounding $\langle \rangle$, and separate the individual **elements** by \cdot 's instead of $,$'s. If σ is a prefix, then $\sigma.n$ is $\sigma \frown \langle n \rangle$; e.g., if $\sigma = 1.2.1$, then $\sigma.3$ is $1.2.1.3$. So for instance,

$$1.2 \mathbb{T} \Box \varphi \rightarrow \varphi$$

is a *prefixed signed formula* (or just a *prefixed formula* for short).

Intuitively, the prefix names a world in a model that might satisfy the **formulas** on a branch of a **tableau**, and if σ names some world, then $\sigma.n$ names a world accessible from (the world named by) σ .

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Bibliography