

## frd.1 Introduction

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One question that interests modal logicians is the relationship between the accessibility relation and the truth of certain **formulas** in models with that accessibility relation. For instance, suppose the accessibility relation is reflexive, i.e., for every  $w \in W$ ,  $Rww$ . In other words, every world is accessible from itself. That means that when  $\Box\varphi$  is true at a world  $w$ ,  $w$  itself is among the accessible worlds at which  $\varphi$  must therefore be true. So, if the accessibility relation  $R$  of  $\mathfrak{M}$  is reflexive, then whatever world  $w$  and formula  $\varphi$  we take,  $\Box\varphi \rightarrow \varphi$  will be true there (in other words, the schema  $\Box p \rightarrow p$  and all its substitution instances are true in  $\mathfrak{M}$ ).

The converse, however, is false. It's not the case, e.g., that if  $\Box p \rightarrow p$  is true in  $\mathfrak{M}$ , then  $R$  is reflexive. For we can easily find a non-reflexive model  $\mathfrak{M}$  where  $\Box p \rightarrow p$  is true at all worlds: take the model with a single world  $w$ , not accessible from itself, but with  $w \in V(p)$ . By picking the truth value of  $p$  suitably, we can make  $\Box\varphi \rightarrow \varphi$  true in a model that is not reflexive.

The solution is to remove the variable assignment  $V$  from the equation. If we require that  $\Box p \rightarrow p$  is true at all worlds in  $\mathfrak{M}$ , regardless of which worlds are in  $V(p)$ , then it is necessary that  $R$  is reflexive. For in any non-reflexive model, there will be at least one world  $w$  such that not  $Rww$ . If we set  $V(p) = W \setminus \{w\}$ , then  $p$  will be true at all worlds other than  $w$ , and so at all worlds accessible from  $w$  (since  $w$  is guaranteed not to be accessible from  $w$ , and  $w$  is the only world where  $p$  is false). On the other hand,  $p$  is false at  $w$ , so  $\Box p \rightarrow p$  is false at  $w$ .

This suggests that we should introduce a notation for model structures without a valuation: we call these *frames*. A frame  $\mathfrak{F}$  is simply a pair  $\langle W, R \rangle$  consisting of a set of worlds with an accessibility relation. Every model  $\langle W, R, V \rangle$  is then, as we say, *based on* the frame  $\langle W, R \rangle$ . Conversely, a frame determines the class of models based on it; and a class of frames determines the class of models which are based on any frame in the class. And we can define  $\mathfrak{F} \models \varphi$ , the notion of a **formula** being *valid* in a frame as:  $\mathfrak{M} \models \varphi$  for all  $\mathfrak{M}$  based on  $\mathfrak{F}$ .

With this notation, we can establish correspondence relations between **formulas** and classes of frames: e.g.,  $\mathfrak{F} \models \Box p \rightarrow p$  if, and only if,  $\mathfrak{F}$  is reflexive.

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## Bibliography