**Definition fil.1.** A system $\Sigma$ of modal logic is said to have the finite model property if whenever a formula $\varphi$ is true at a world in a model of $\Sigma$ then $\varphi$ is true at a world in a finite model of $\Sigma$.

**Proposition fil.2.** $K$ has the finite model property.

*Proof.* $K$ is the set of valid formulas, i.e., any model is a model of $K$. By $\varphi$ if $M, w \models \varphi$, then $M^\ast, w \models \varphi$ for any filtration of $M$ through the set $\Gamma$ of sub-formulas of $\varphi$. Any formula only has finitely many sub-formulas, so $\Gamma$ is finite. By $\varphi$, $|W^\ast| \leq 2^n$, where $n$ is the number of formulas in $\Gamma$. And since $K$ imposes no restriction on models, $M^\ast$ is a $K$-model.

To show that a logic $L$ has the finite model property via filtrations it is essential that the filtration of an $L$-model is itself a $L$-model. Often this requires a fair bit of work, and not any filtration yields a $L$-model. However, for universal models, this still holds.

**Proposition fil.3.** Let $\mathcal{U}$ be the class of universal models (see $??$) and $\mathcal{U}_{\text{Fin}}$ the class of all finite universal models. Then any formula $\varphi$ is valid in $\mathcal{U}$ if and only if it is valid in $\mathcal{U}_{\text{Fin}}$.

*Proof.* Finite universal models are universal models, so the left-to-right direction is trivial. For the right-to-left direction, suppose that $\varphi$ is false at some world $w$ in a universal model $M$. Let $\Gamma$ contain $\varphi$ as well as all of its subformulas; clearly $\Gamma$ is finite. Take a filtration $M^\ast$ of $M$; then $M^\ast$ is finite by $??$, and by $??$, $\varphi$ is false at $[w]$ in $M^\ast$. It remains to observe that $M^\ast$ is also universal: given $u$ and $v$, by hypothesis $Ruv$ and by $??$, also $R^\ast[u][v]$.

**Corollary fil.4.** $S5$ has the finite model property.

*Proof.* By $??$, if $\varphi$ is true at a world in some reflexive and euclidean model then it is true at a world in a universal model. By Proposition fil.3, it is true at a world in a finite universal model (namely the filtration of the model through the set of sub-formulas of $\varphi$). Every universal model is also reflexive and euclidean; so $\varphi$ is true at a world in a finite reflexive euclidean model.

**Problem fil.1.** Show that any filtration of a serial or reflexive model is also serial or reflexive (respectively).

**Problem fil.2.** Find a non-symmetric (non-transitive, non-euclidean) filtration of a symmetric (transitive, euclidean) model.