

prf.1 More Proofs in \mathbf{K}

[nml:prf:mpr:sec](#) Let's see some more examples of **derivability** in \mathbf{K} , now using the simplified method introduced in ??.

Proposition prf.1. $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$

Proof.

1. $\mathbf{K} \vdash (\varphi \rightarrow \psi) \rightarrow (\neg\psi \rightarrow \neg\varphi)$ PL
2. $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\Box\neg\psi \rightarrow \Box\neg\varphi)$ RK, 1
3. $\mathbf{K} \vdash (\Box\neg\psi \rightarrow \Box\neg\varphi) \rightarrow (\neg\Box\neg\varphi \rightarrow \neg\Box\neg\psi)$ TAUT
4. $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\neg\Box\neg\varphi \rightarrow \neg\Box\neg\psi)$ PL, 2, 3
5. $\mathbf{K} \vdash \Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$ \Diamond for $\neg\Box\neg$. □

Proposition prf.2. $\mathbf{K} \vdash \Box\varphi \rightarrow (\Diamond(\varphi \rightarrow \psi) \rightarrow \Diamond\psi)$

Proof.

1. $\mathbf{K} \vdash \varphi \rightarrow (\neg\psi \rightarrow \neg(\varphi \rightarrow \psi))$ TAUT
2. $\mathbf{K} \vdash \Box\varphi \rightarrow (\Box\neg\psi \rightarrow \Box\neg(\varphi \rightarrow \psi))$ RK, 1
3. $\mathbf{K} \vdash \Box\varphi \rightarrow (\neg\Box\neg(\varphi \rightarrow \psi) \rightarrow \neg\Box\neg\psi)$ PL, 2
4. $\mathbf{K} \vdash \Box\varphi \rightarrow (\Diamond(\varphi \rightarrow \psi) \rightarrow \Diamond\psi)$ \Diamond for $\neg\Box\neg$. □

Proposition prf.3. $\mathbf{K} \vdash (\Diamond\varphi \vee \Diamond\psi) \rightarrow \Diamond(\varphi \vee \psi)$

Proof.

1. $\mathbf{K} \vdash \neg(\varphi \vee \psi) \rightarrow \neg\varphi$ TAUT
2. $\mathbf{K} \vdash \Box\neg(\varphi \vee \psi) \rightarrow \Box\neg\varphi$ RK, 1
3. $\mathbf{K} \vdash \neg\Box\neg\varphi \rightarrow \neg\Box\neg(\varphi \vee \psi)$ PL, 2
4. $\mathbf{K} \vdash \Diamond\varphi \rightarrow \Diamond(\varphi \vee \psi)$ \Diamond for $\neg\Box\neg$
5. $\mathbf{K} \vdash \Diamond\psi \rightarrow \Diamond(\varphi \vee \psi)$ similarly
6. $\mathbf{K} \vdash (\Diamond\varphi \vee \Diamond\psi) \rightarrow \Diamond(\varphi \vee \psi)$ PL, 4, 5. □

Proposition prf.4. $\mathbf{K} \vdash \Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \vee \Diamond\psi)$

Proof.

1. $\mathbf{K} \vdash \neg\varphi \rightarrow (\neg\psi \rightarrow \neg(\varphi \vee \psi))$ TAUT
2. $\mathbf{K} \vdash \Box\neg\varphi \rightarrow (\Box\neg\psi \rightarrow \Box\neg(\varphi \vee \psi))$ RK
3. $\mathbf{K} \vdash \Box\neg\varphi \rightarrow (\neg\Box\neg(\varphi \vee \psi) \rightarrow \neg\Box\neg\psi)$ PL, 2
4. $\mathbf{K} \vdash \neg\Box\neg(\varphi \vee \psi) \rightarrow (\Box\neg\varphi \rightarrow \neg\Box\neg\psi)$ PL, 3
5. $\mathbf{K} \vdash \neg\Box\neg(\varphi \vee \psi) \rightarrow (\neg\neg\Box\neg\psi \rightarrow \neg\Box\neg\varphi)$ PL, 4
6. $\mathbf{K} \vdash \Diamond(\varphi \vee \psi) \rightarrow (\neg\Diamond\psi \rightarrow \Diamond\varphi)$ \Diamond for $\neg\Box\neg$
7. $\mathbf{K} \vdash \Diamond(\varphi \vee \psi) \rightarrow (\Diamond\psi \vee \Diamond\varphi)$ PL, 6. □

Problem prf.1. Show that the following **derivability** claims hold:

1. $\mathbf{K} \vdash \Diamond \neg \perp \rightarrow (\Box \varphi \rightarrow \Diamond \varphi)$;
2. $\mathbf{K} \vdash \Box(\varphi \vee \psi) \rightarrow (\Diamond \varphi \vee \Box \psi)$;
3. $\mathbf{K} \vdash (\Diamond \varphi \rightarrow \Box \psi) \rightarrow \Box(\varphi \rightarrow \psi)$.

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Bibliography