**Def: Duals**

Each of the formulas T, B, 4, and 5 has a *dual*, denoted by a subscripted diamond, as follows:

\[
\begin{align*}
p \rightarrow \lozenge p & \quad (T_\lozenge) \\
\lozenge \Box p \rightarrow p & \quad (B_\lozenge) \\
\lozenge \lozenge p \rightarrow \lozenge p & \quad (4_\lozenge) \\
\lozenge \Box p \rightarrow \Box p & \quad (5_\lozenge)
\end{align*}
\]

Each of the above dual formulas is obtained from the corresponding formula by substituting \( \neg p \) for \( p \), contraposing, replacing \( \neg \Box \neg \) by \( \lozenge \), and replacing \( \neg \lozenge \neg \) by \( \Box \). D, i.e., \( \Box \varphi \rightarrow \lozenge \varphi \) is its own dual in that sense.

**Problem prf.1.** Show that for each formula \( \varphi \) in Definition prf.1: \( K \vdash \varphi \leftrightarrow \varphi_\lozenge \).

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**Bibliography**