bas.1 Overspill

mod:bas:ove:

mod:bas:ove: Theorem bas.1. If a set Γ of sentences has arbitrarily large finite models, overspill then it has an infinite model.

Proof. Expand the language of Γ by adding countably many new constants c_0 , c_1, \ldots and consider the set $\Gamma \cup \{c_i \neq c_j : i \neq j\}$. To say that Γ has arbitrarily large finite models means that for every m > 0 there is $n \geq m$ such that Γ has a model of cardinality n. This implies that $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ is finitely satisfiable. By compactness, $\Gamma \cup \{c_i \neq c_j : i \neq j\}$ has a model \mathfrak{M} whose domain must be infinite, since it satisfies all inequalities $c_i \neq c_j$.

mod:bas:ove: Proposition bas.2. There is no sentence φ of any first-order language that is inf-not-fo true in a structure \mathfrak{M} if and only if the domain $|\mathfrak{M}|$ of the structure is infinite.

Proof. If there were such a φ , its negation $\neg \varphi$ would be true in all and only the finite structures, and it would therefore have arbitrarily large finite models but it would lack an infinite model, contradicting Theorem bas.1.

Photo Credits

Bibliography